Purushottam School of Engineering & Technology, Rourkela

4th seme mechanical

Fluied mechanics

STATE COUNCIL FOR TECHNICAL EDUCATION AND VOCATIONAL TRAINING, ODISHA									
TEACHING AND EVALUATION SCHEME FOR 4th Semester (Mechanical Engg.) (wef. 2019-20)									
Subject Number	Subject Code	Subject	Periods/week			Evaluation Scheme			
			L	T	P	Internal Assessment/ Sessional	End Sem Exams	Exams (Hours)	Total
		Theory							
Th.1		Theory of Machine	4		-	20	80	3	100
Th.2		Manufacturing Technology	4		-	20	80	3	100
Th.3		Fluid Mechanics	4		-	20	80	3	100
Th.4		Thermal Engg-II	4		-	20	80	3	100
		Total	*16*			*80*	*320*	-	*400*
		Practical							
Pr.1		Theory of Machine and Measurement lab	-	-	6	25	75	3	100
Pr.2		Mechanical Engg. Lab-II	-	-	6	25	75	3	100
Pr.3		Workshop-III	-	-	6	50	50	4	100
Pr.4		Technical Seminar			2	50			50
		Student Centered Activities(SCA)		-	3				
		Total	-	-	23	150	200	-	350
		Grand Total	**16**	**-**	**23**	**230**	**520**	**-**	**750**
Abbreviations: L-Lecturer, T-Tutorial, P-Practical . Each class is of minimum 55 minutes duration									
Minimum Pass Mark in each Theory subject is 35% and in each Practical subject is 50% and in Aggregate is 40%									
SCA shall comprise of Extension Lectures/ Personality Development/ Environmental issues /Quiz /Hobbies/ Field visits/ cultural activities/Library studies/Classes on MOOCS/SWAYAM etc. ,Seminar and SCA shall be conducted in a section.									

CURRICULLUM OF 4th SEMESTER

For

DIPLOMA IN MECHANICAL ENGINEERING

(Effective FROM 2019-20 Sessions)

STATE COUNCIL FOR TECHNICAL EDUCATION & VOCATIONAL TRAINING, ODISHA, BHUBANESWAR

TH-3 FLUID MECHANICS

Name of the Course: Diploma in **Mech & Other Mechanical Allied Branches**			
Course code:		Semester	4th
Total Period:	60	Examination	3 hrs
Theory periods:	4 P/W	Class Test:	20
Maximum marks:	100	End Semester Examination:	80

A. RATIONAL:

Use of fluid in engineering field is of great importance. It is therefore necessary to study the physical properties and characteristics of fluids which have very important application in mechanical and automobile engineering.

B. COURSE OBJECTIVES:

Students will develop an ability towards

- Comprehending fluid properties and their measurements
- Realizing conditions for floatation
- Applying Bernoulli's theorem

C. TOPIC WISE DISTRIBUTION OF PERIODS

Sl. No.	Topic	Periods
01	Properties of Fluid	08
02	Fluid Pressure and its measurements	08
03	Hydrostatics	08
04	Kinematics of Flow	08
05	orifices, notches & weirs	08
06	Flow through pipe	10
07	Impact of jets	10
	Total Period:	**60**

D.CONTENT

1.0 Properties of Fluid

1.1 Define fluid

1.2 Description of fluid properties like Density, Specific weight, specific gravity, specific volume and solve simple problems.

1.3 Definitions and Units of Dynamic viscosity, kinematic viscosity, surface tension Capillary phenomenon

2.0 Fluid Pressure and its measurements

2.1 Definitions and units of fluid pressure, pressure intensity and pressure head.

2.2 Statement of Pascal's Law.

2.3 Concept of atmospheric pressure, gauge pressure, vacuum pressure and absolute pressure

2.4 Pressure measuring instruments

Manometers (Simple and Differential)

2.4.1 Bourdon tube pressure gauge(Simple Numerical)

2.5 Solve simple problems on Manometer.

3.0 Hydrostatics

3.1 Definition of hydrostatic pressure

3.2 Total pressure and centre of pressure on immersed bodies(Horizontal and Vertical Bodies)

3.3 Solve Simple problems.

3.4 Archimedes 'principle, concept of buoyancy, meta center and meta centric height (Definition only)

3.5 Concept of floatation

4.0 Kinematics of Flow

4.1 Types of fluid flow

4.2 Continuity equation(Statement and proof for one dimensional flow)

4.3 Bernoulli's theorem(Statement and proof)
Applications and limitations of Bernoulli's theorem (Venturimeter, pitot tube)

4.4 Solve simple problems

5.0 Orifices, notches & weirs

5.1 Define orifice

5.2 Flow through orifice

5.3Orifices coefficient & the relation between the orifice coefficients

5.4 Classifications of notches & weirs

5.5 Discharge over a rectangular notch or weir

5.6 Discharge over a triangular notch or weir

5.7 Simple problems on above

6.0 Flow through pipe

6.1 Definition of pipe.

6.2 Loss of energy in pipes.

6.3 Head loss due to friction: Darcy's and Chezy's formula (Expression only)

6.4 Solve Problems using Darcy's and Chezy's formula.

6.5 Hydraulic gradient and total gradient line

7.0 Impact of jets

7.1 Impact of jet on fixed and moving vertical flat plates

7.2 Derivation of work done on series of vanes and condition for maximum efficiency.

7.3 Impact of jet on moving curved vanes, illustration using velocity triangles, derivation of work done, efficiency.

CHAPTERS COVERED UP TO IA- 1, 2,3,4

Learning Resources:

Sl No.	Name of the Book	Author Name	Publisher
1.	Text Book of Fluid Mechanics	R.K.Bansal	Laxmi
2.	Text Book of Fluid Mechanics	R.S khurmi	S.Chand
3.	Text Book of Fluid Mechanics	R.K.Rajput	S.Chand
4.	Text Book of Fluid Mechanics	Modi & Seth	Rajson's pub. Pvt. lt

FM

Mechanics
(study of motion)

Kinematics — study of motion without the consideration of Basic cause of motion i.e. force.

Dynamics — study of motion withou the consideration of basic cause of motion i.e. force.

$\bar{V} = \frac{d\bar{s}}{dt}$

$\bar{a} = \frac{d\bar{v}}{dt}$

$\bar{J} = \frac{d\bar{a}}{dt}$ (jerk)

Newton's Second low.

$\vec{F_{ext}} = \frac{d}{dt}(m\bar{v})$ → Mass

$\vec{F_{ext}} = m \cdot \frac{d\bar{v}}{dt}$

$= m \cdot a$

where a - acceleration (pure kinematic quantity)

Dynamic viscosity (μ) - $N \cdot s/m^2$

involving mass quanity - Dynamic.

Kinematic viscosity (ν) - m^2/sec

(no mass term. Kinematic).

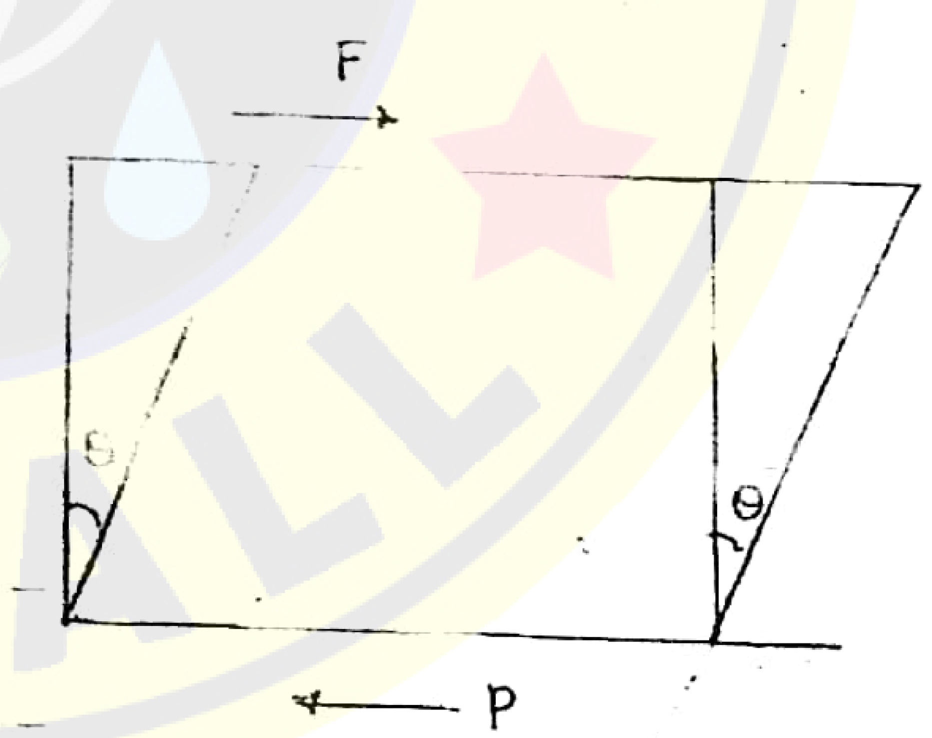
Fluid :

" liquids and gases both are having a property of continuous deformation under the action of shear or tangential forces whereas this property is not inhereted in solids. This property of the continuous deformation is known as flow property. and just because of having this flow property, liquids and gases both are kept in different category which is far away from solids, known as Fluid."

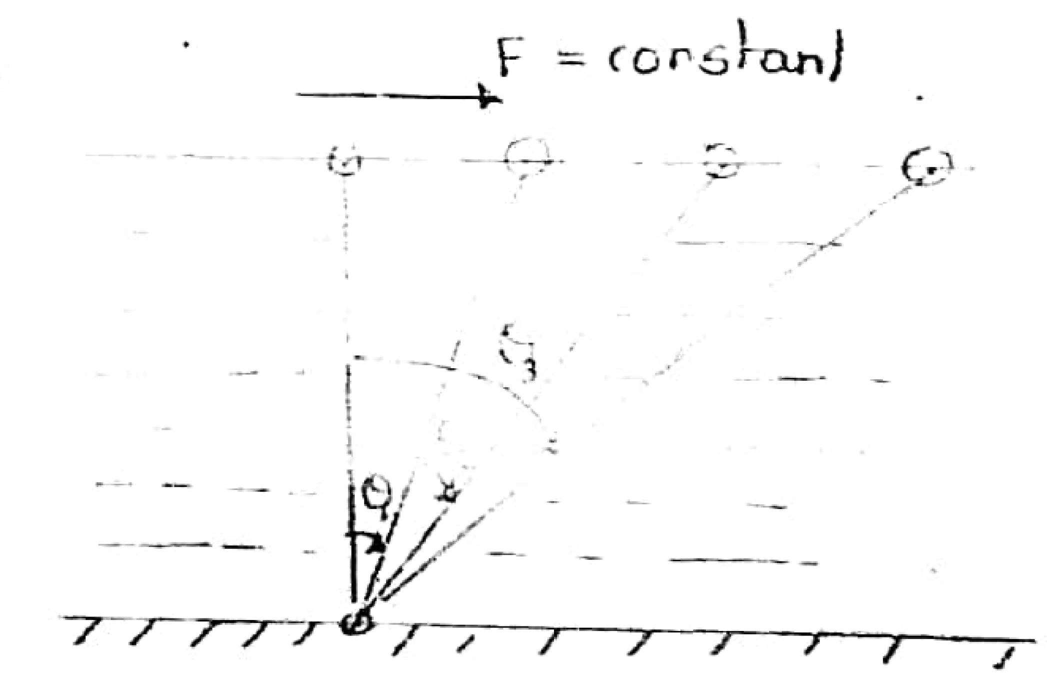
" A fluid is substance which is ~~having~~ an ability to flow under the action of shear or tangential forces."

Matter — solids, liquids, gases

liquids, gases] Fluids.



Solids - fixed deformation (θ)



liquids / gases - continuous deformation under action of const. force - flow

Fluid as Continuum :

"In macrosystems, the intermolecular distances in liquids and gases can be treated as negligible, as compared to the dimensions of the system. Therefore, the entire fluid mass system can be treated as the continuous distribution of mass and this continuous mass of fluid is known as Continuum."

Basic properties of Fluid :

1. Density (ρ) :

"It is defined as mass per unit volume of the substance."

$$\rho = \frac{m}{\forall} \qquad \forall - \text{volume}$$

MKS unit : kg/m^3

g/cm^3 or g/cc

$$1\ g/cc = \frac{10^{-3}\ kg}{10^{-6}\ m^3} = 10^3\ kg/m^3$$

Density of water (ρ) = $1000\ kg/m^3$

Density of air (at atmospheric conditions) = $1.21\ kg/m^3$

2. Specific weight :

"It is defined as the weight of the substance per unit volume."

$$\text{sp. weight} = \frac{m \cdot g}{\forall} = \rho \cdot g \quad N/m^3$$

3. Specific gravity :

"It is defined as ratio of density of fluid to the density of standard fluid."

$$(\text{Sp. gravity})_{\text{fluid}} = \frac{\text{Density of fluid}}{\text{Density of standard fluid}}$$

For liquids, standard fluid is water ($\rho = 1000\ kg/m^3$)

For gases, standard fluid is atmospheric air ($\rho = 1.21\ kg/m^3$)

Sometimes H_2 or CH_4 are taken as standard fluids for gases.

4. Relative density :

$$(R.D.)_{1/2} = \frac{\text{Density of } 1^{st} \text{ fluid}}{\text{Density of } 2^{nd} \text{ fluid}} = \frac{\rho_1}{\rho_2}$$

Relative density of fluid is always defined w.r.t. the another fluid.

5. Compressibility :

$$\beta = \frac{-\left(\frac{d\forall}{\forall}\right)}{dP}$$

Mass, m = constant

$\rho \cdot \forall$ = constant

$$\rho\, d\forall + \forall\, d\rho = 0$$

$$\frac{-d\forall}{\forall} = \frac{d\rho}{\rho}$$

$$\beta = \frac{1}{\rho}\frac{d\rho}{dP} \qquad m^2/N$$

If density ρ is not changing w.r.t. P then,

$$\frac{d\rho}{dP} = 0 \qquad \beta = 0$$

∴ The fluids are called Incompressible

If density ρ is changing w.r.t. P then,

$$\frac{d\rho}{dP} \neq 0. \qquad \beta \neq 0$$

∴ The fluids are called Compressible.

(i) For liquids.

(practically compressible)

consider

Water (at 1 atm pressure) — 998 kg/m^3

Water (at 100 atm pressure) — 1003 kg/m^3

These are actual values of densities of water at normal temperature condition.

$$\Delta\rho = 5 \ kg/m^3$$

$$\% \text{ change in density} = \frac{5}{998} \times 100 \simeq 0.5\%$$

0.5% is very small thus considered zero i.e. $\beta = 0$

Just because $\beta \to 0$ liquids are treated as the Incompressible fluids.

For gases

(highly compressible)

Ideal gas equation

$$PV = mRT$$

$$P = \rho RT$$

$$\rho \propto P \qquad \text{at same temperature.}$$

$$\text{Mach number } (M_a) = \frac{V_{object}}{V_{sound}} = \frac{V_{object}}{C}$$

If Mach number < 0.3, gases are treated as (for incompressible.

Hints

Note: The reciprocal of compressibility is known as bulk modulus of elasticity (k)

$$k = \frac{1}{\beta}$$

compressibility is defined ... and then bulk modulus is defined using compressibility.

6. Isothermal compressibility of gas:

$$\beta = \frac{1}{\rho}\frac{d\rho}{dP}$$

from Ideal gas equation.

$$P = \rho RT$$

$$\rho = \frac{P}{RT} \qquad \text{(T- constant for the Isothermal process)}$$

$$d\rho = \frac{1}{RT}\cdot dP$$

$$\frac{d\rho}{dP} = \frac{1}{RT}$$

$$\beta = \frac{1}{\rho} \times \frac{1}{RT}$$

$$= \frac{1}{P} \qquad \because \rho RT = P$$

$$(\beta)_{Isothermal} = \frac{1}{P}$$

$$(k)_{Isothermal} = P$$

7. Adiabatic compressibility of gas:

$$\beta = \frac{1}{\rho} \frac{d\rho}{dP}$$

Adiabatic equation.

$$P \forall^{\gamma} = \text{constant}$$

γ - adiabatic exponent.

For air (di-atomic)

$\gamma = 1.4$

$$P \cdot \frac{m^{\gamma}}{\rho^{\gamma}} = \text{constant}$$

$$P \cdot \rho^{-\gamma} = \text{constant} \qquad (\because m - \text{constant})$$

$$P(-\gamma) \rho^{-\gamma-1} d\rho + \rho^{-\gamma} dP = 0$$

$$\frac{d\rho}{dP} = \frac{\rho}{\gamma \cdot P}$$

$$\beta = \frac{1}{\rho} \cdot \frac{\rho}{\gamma \cdot P}$$

$$(\beta)_{Adiabatic} = \frac{1}{\gamma P}$$

$$(k)_{Adiabatic} = \gamma \cdot P$$

Newton's failure:

velocity of sound

in solids. $v = \sqrt{\frac{E}{\rho}}$

in liquids $C = \sqrt{\frac{K}{\rho}}$

in gases $C = \sqrt{\frac{P}{\rho}}$, which is wrong

$\because$ The atmospheric conditions on earth are adiabatic, the $(\gamma \cdot P)$ has to be used.

$\therefore$ Actual velocity of sound in gases is

$$C = \sqrt{\frac{\gamma \cdot P}{\rho}} \qquad \text{(Laplace's correction)}$$

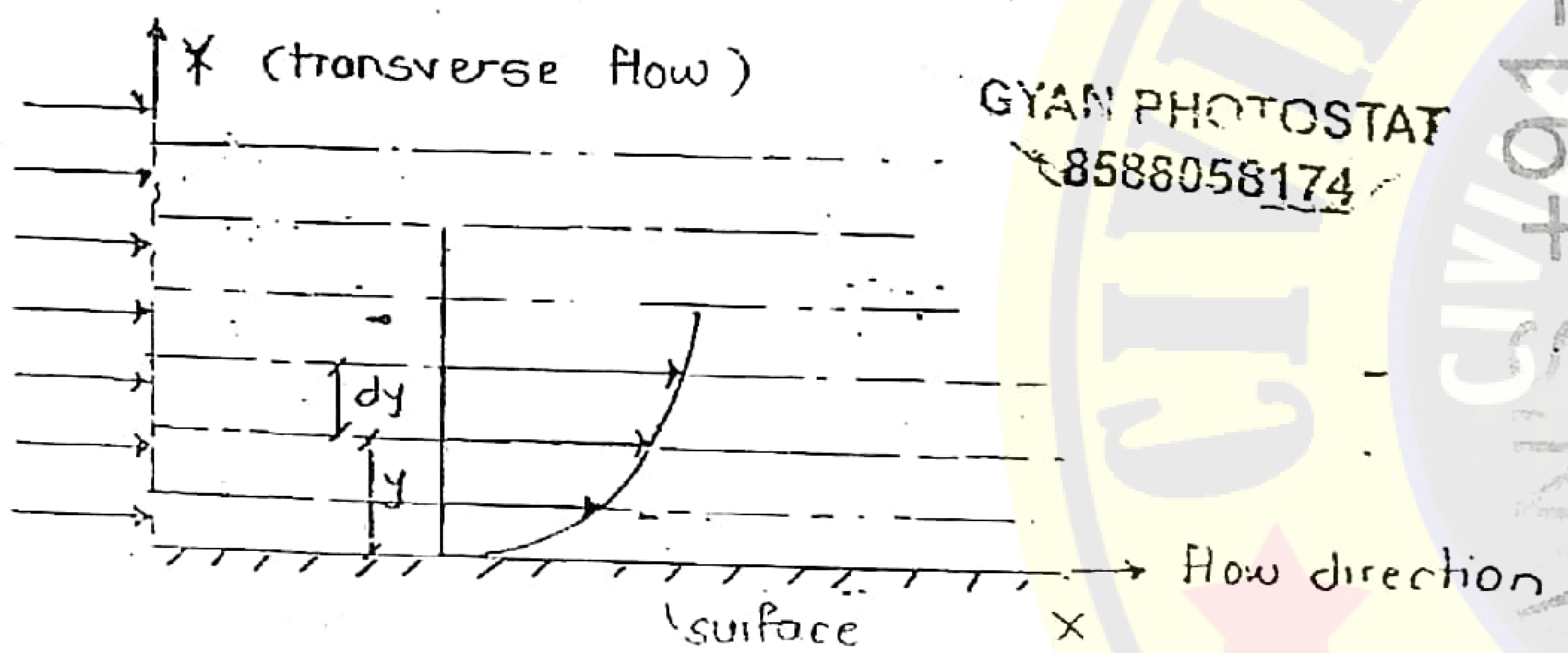
Viscosity :

"The two adjacent layers of a fluid resist the motion of each other. Such a fundamental property of fluid is known as Viscosity."

It is also known as the internal friction between the layers of the fluid.

Basic cause of viscosity :

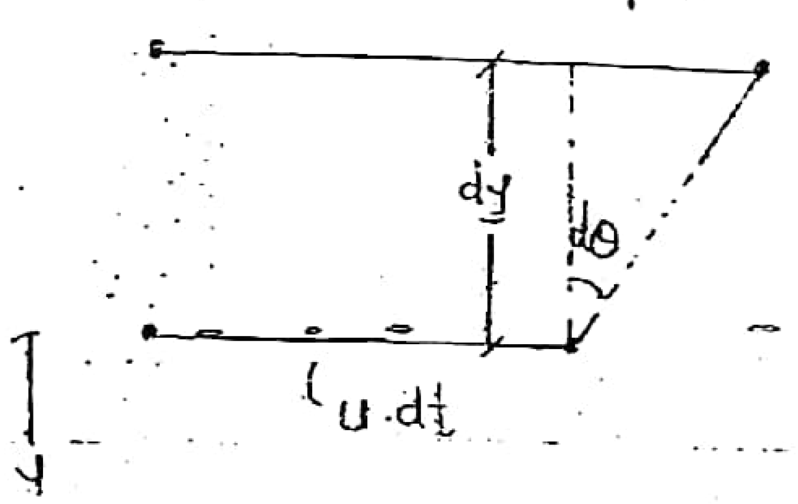
The basic cause of viscosity is the cohesive forces between the molecules of fluid i.e. cohesion.



The relative velocity of contacting layer with the surface is zero (No slip condition)

There will be the development of velocity gradient in transverse direction of flow.

velocity gradient - $\left(\frac{\partial u}{\partial y}\right)$



$$\tan d\theta = \frac{du.dt}{dy}$$

$$d\theta = \frac{du.dt}{dy}$$

dθ - angular deformation (shear

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

Rate of shear deformation.

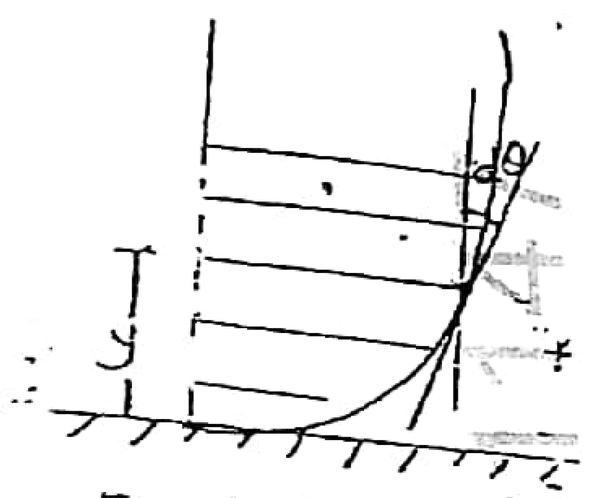


Fig. Velocity grad

Newton's law of viscosity :

The viscous shear stress between the two adjacent layers of fluid at a distance y from surface is

$$\tau \propto \frac{d\theta}{dt}$$ (only for Laminar flows)

$$\tau = \mu \frac{d\theta}{dt}$$

μ - constant (not universal) but it is property of fluid and it also depends on temperature.

$$\mu = \frac{\tau}{\left(\frac{d\theta}{dt}\right)}$$

If μ is high, $\frac{d\theta}{dt}$ is less i.e. flow is difficult

If μ is less, $\frac{d\theta}{dt}$ is high i.e. flow is easy.

It means,

μ is the direct measurement of internal resistance between the layers of fluids.

μ is called Dynamic viscosity.

Units of μ :

$$\mu = \frac{\tau}{\left(\frac{d\theta}{dt}\right)} = \frac{\tau'}{\left(\frac{du}{dy}\right)}$$

S.I. unit :

$$\frac{N\cdot s}{m^2} = Pa.s.$$

$$1\ Pa.s = 1.\ \frac{N\cdot s}{m^2}$$

MKS unit :

$$1\ Pa\cdot s = 1.\ \frac{Ns}{m^2}$$

$$= 1\ \frac{kg\cdot m}{s^2}\cdot\frac{s}{m^2}$$

$$1\cdot Pa\cdot s = 1\ \frac{kg}{m\cdot s}$$

C.G.S. unit : (Poise) :

$$1\ Poise = 1\ \frac{gm}{cm\cdot s}$$

$$= \frac{10^{-3}\ kgm}{10^{-2}\ m\cdot s}$$

$$= 0.1\ \frac{kg}{m\cdot s}$$

$$1\ Poise = 0.1\ Pa\cdot s$$

Units of γ :

$$\gamma = \frac{\mu}{\rho}$$

M.K.S. unit: m^2/s

CGS unit (stokes)

Effect of temperature on viscosity of fluid :

The main reason of viscosity in fluids is the cohesion. But in gases, the cohesion is almost nill. Thus, in gases, particles are in random motion (Brownian mo -given by Robert Brown)

$$(\mu)_{gas} <<< (\mu)_{liquids}$$

e.g.

$$\mu_{water} = 55\ \mu_{gas\ i.e.\ air.}$$

We know that.

$$\gamma_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$$

Kinematic viscosity of gas may be more than the dynamic viscosity, equal to dynamic viscosity or less than dynamic viscosity depending upon temperature and pressure conditions.

e.g. $\quad \gamma_{air\,(atm)} > \gamma_{water}$

Thus Absolute viscosity of gas is dynamic viscosity.

If temperature increases, thermal expansion takes place. thus, density of gas decreases

But $\quad \rho_{liq}$ decreases very slightly

& $\quad \rho_{gas}$ decrease very highly

Becaus.

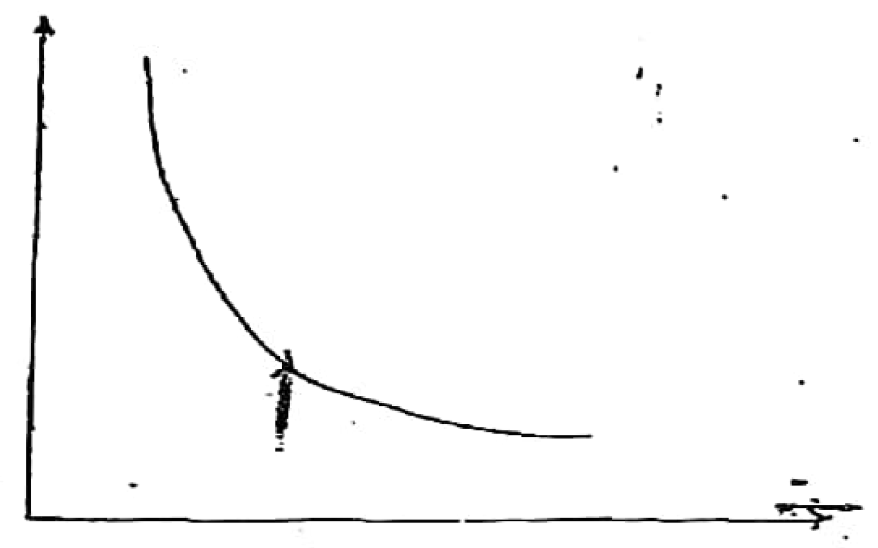
$$P = \rho RT.$$

$$\rho_{gas} = \frac{P}{RT}$$

For liquids:

If temperature increases cohesion decreases and dynamic viscosity also decreases

viscosity (μ)



Temperature →

If temperature increases, kinematic viscosity also decreases but with very slight decrease

because μ decreases

but $(\rho)_{liq}$ decreases very slightly

$$\nu_{liq} = \frac{\mu_{liq}}{\rho_{liq}}$$

Thus decrease in dynamic viscosity is more than the kinematic viscosity

For gases:

Cohesion in gases is almost zero

By Kinematic theory of gases.

$$C_{rms} = \sqrt{\frac{3RT}{M}}$$

$$C_{rms} \propto \sqrt{T}$$

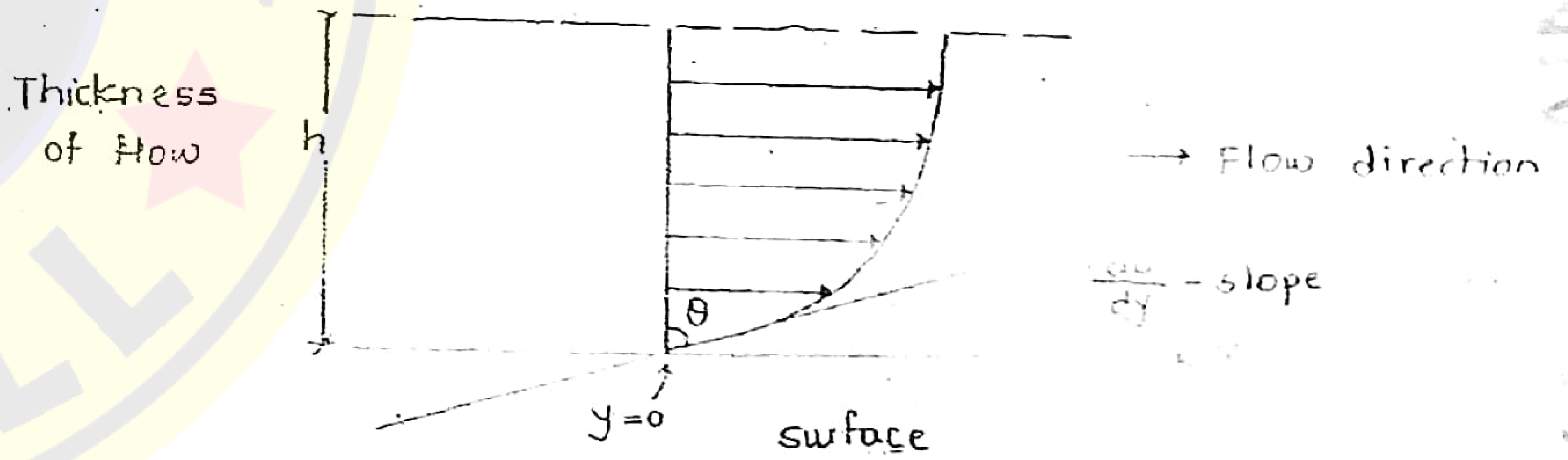
i.e. If temperature increases, root mean square velocity of a gas increases also.

Thus, the randomness in the motion of gas molecules will increase. It will introduce some additional resistance in the path of fluid flow. (i.e. viscosity)

$$\nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$$

Due to increase in temperature dynamic viscosity of gas (μ_{gas}) and kinematic viscosity of gas (ν_{gas}) both increases but increase in ν_{gas} is more as compared to μ_{gas}

Linearisation of Newton's law of viscosity:



According to Newton's law of viscosity.

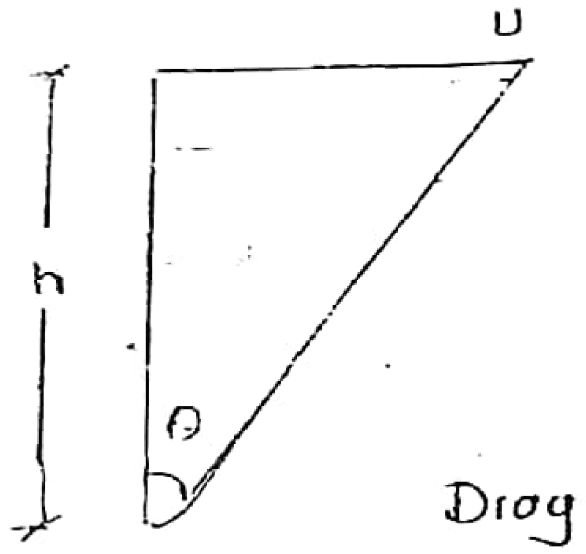
$$\tau = \mu \frac{du}{dy}$$

At surface.

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{at\ y=0}$$

To find $\left(\frac{du}{dy}\right)_{at\ y=0}$ (velocity profile is unknown)

If thickness of flow is very very small i.e. of the order of mm Then velocity profile can be treated



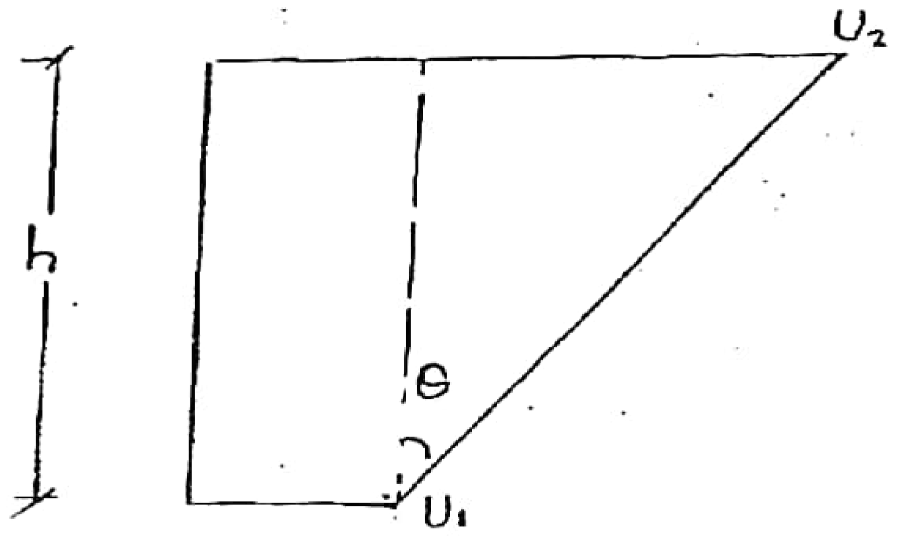
$$\tau_o = \mu \left[\frac{u-0}{h}\right]$$

$$\tau_o = \frac{\mu u}{h}$$

Drag force, $F_D = \tau_o A$

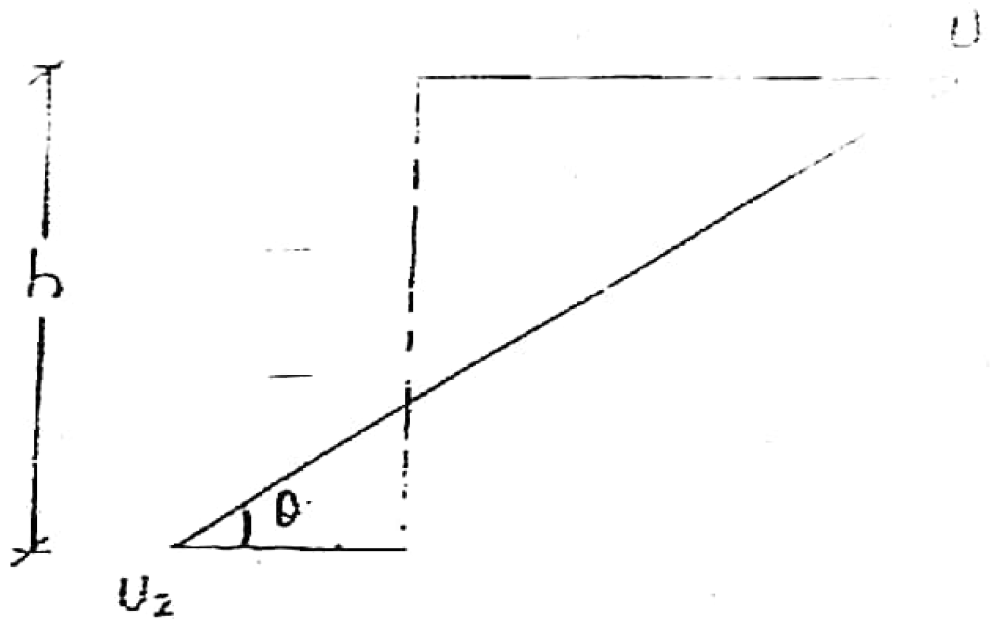
$$= \frac{\mu \cdot u}{h} \cdot A$$ (Drag force on the surface by fluid)

If plate placed on fluid is moving forward



$$\tau = \mu \left[\frac{U_2 - U_1}{h}\right]$$

If plate placed on fluid is moving in opposite direction to fluid flow

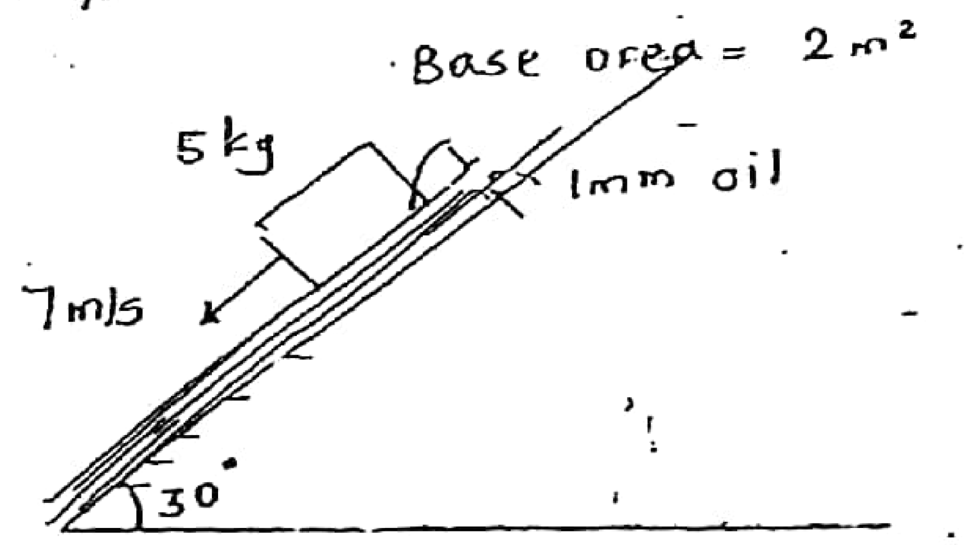


$$\tau = \mu \left[\frac{U_1 - (-U_2)}{h}\right]$$

Drag force is applied on the surface by fluid while the viscous force is force between the layers of fluids.

Q. Find the viscosity.

2 Marks



As velocity is constant, net force is zero.

$$F = 0$$

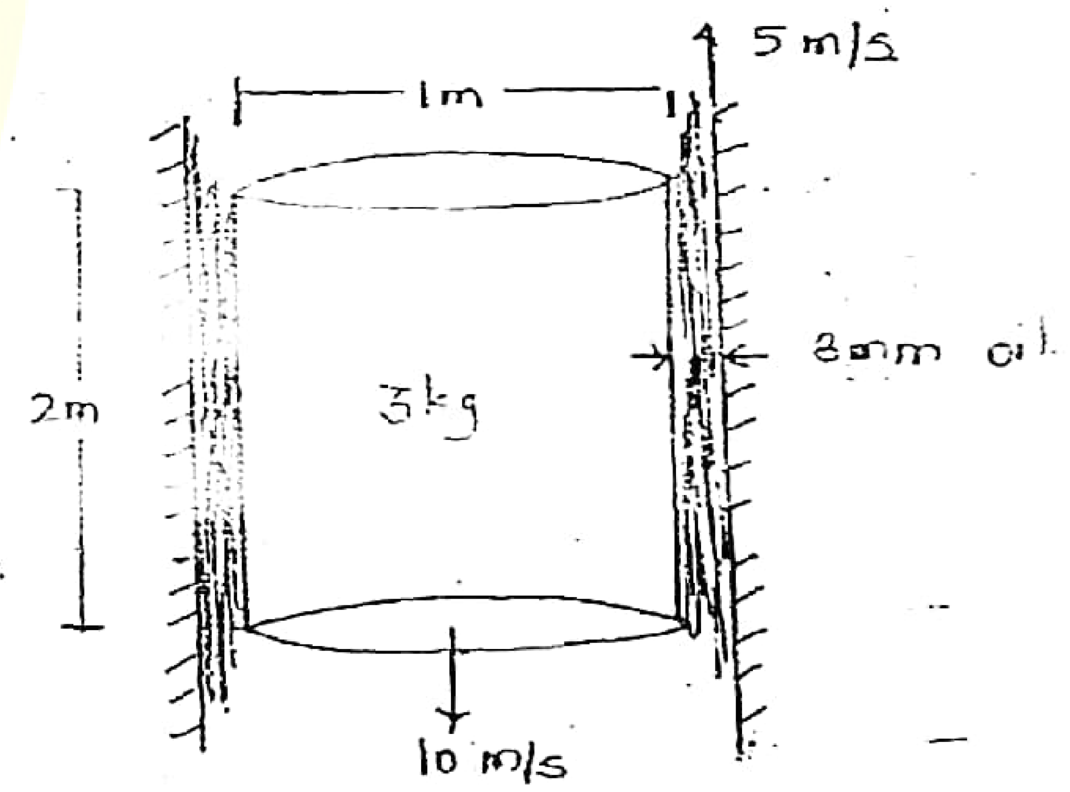
$$mg \cdot \sin\theta = F_D$$

$$5 \times 9.81 \times \sin 30^\circ = \mu \left(\frac{7-0}{1\times10^{-3}}\right) \cdot 2$$

$$\mu = 1.75 \times 10^{-3} \ Ns/m^2$$

Q. Find dynamic viscosity of oil.

2 Marks



As v is constant.

$$F = 0$$

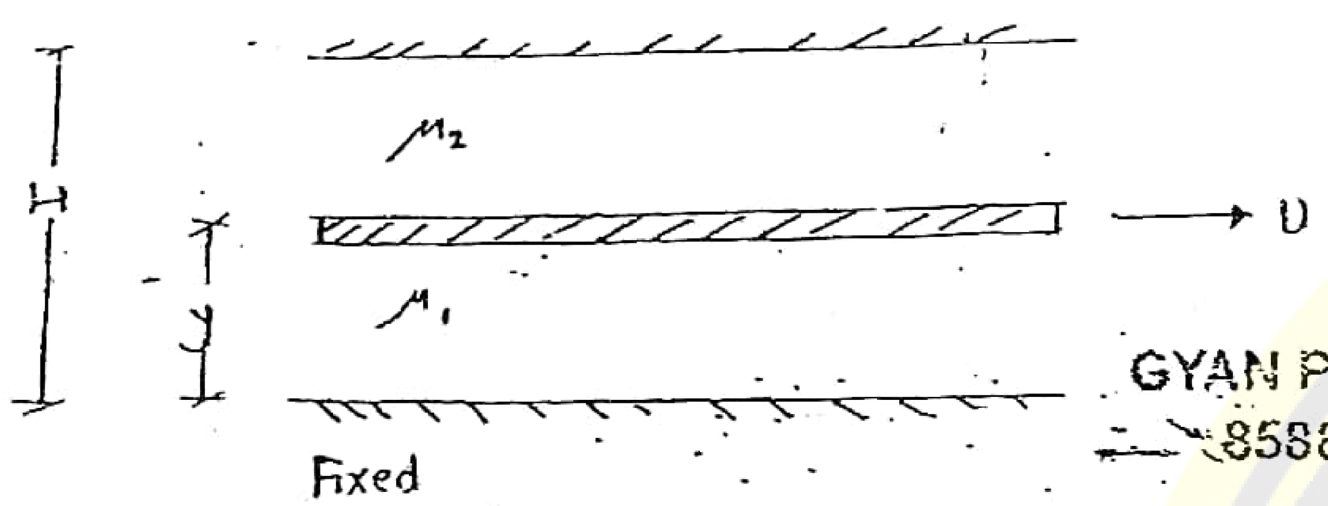
$$mg = F_D$$

$$3 \times 9.81 = \mu \left[\frac{10-(-5)}{3\times10^{-3}}\right] \times (2\pi \times 1 \times 2)$$

$$\mu = 4.68 \times 10^{-4} \ Ns/m^2$$

Q. Find (i) y such that drag on moving plate by both fluids is same
(ii) y such that total drag is minimum.

10 Marks



(i) Drag on both sides is same

$$F_{D_1} = F_{D_2}$$

$$\mu_1 \left[\frac{U-0}{y}\right] \cdot A = \left[\frac{U-0}{H-y}\right] \cdot \mu_2 \cdot A$$

$$\frac{\mu_1}{y} = \frac{\mu_2}{H-y}$$

$$y = \left(\frac{\mu_1}{\mu_1 + \mu_2}\right) H$$

(ii) For minimum drag.

$$F_D = F_{D_1} + F_{D_2}$$

$$= \frac{\mu_1 UA}{y} + \frac{\mu_2 UA}{H-y}$$

$$F_D = UA\left[\frac{\mu_1}{y} + \frac{\mu_2}{H-y}\right]$$

for drag to be minimum,

$$(F_D)_{min} = \frac{dF_D}{dy} = 0$$

$$UA\left[-\frac{\mu_1}{y^2} + \frac{\mu_2}{(H-y)^2}\right] = 0$$

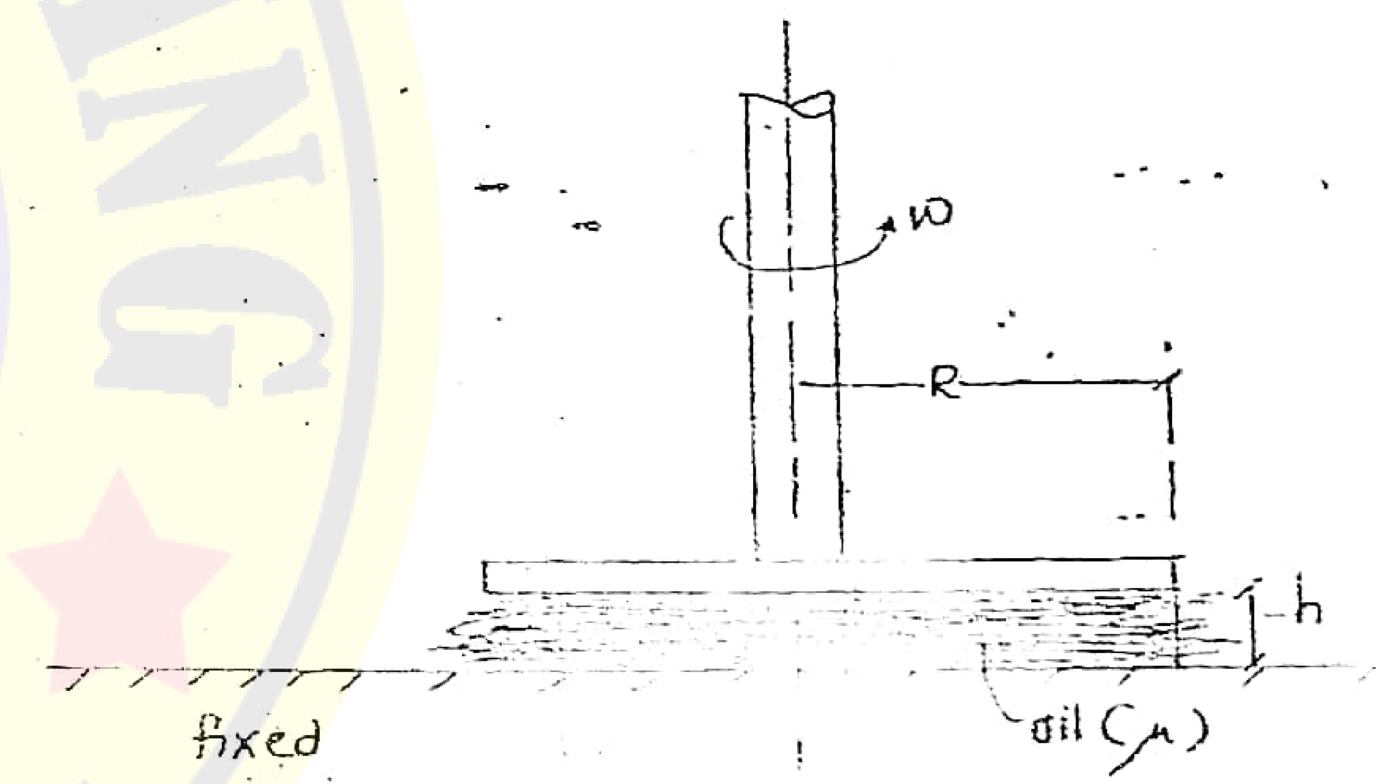
$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(H-y)^2}$$

$$\frac{\sqrt{\mu_1}}{y} = \frac{\sqrt{\mu_2}}{(H-y)}$$

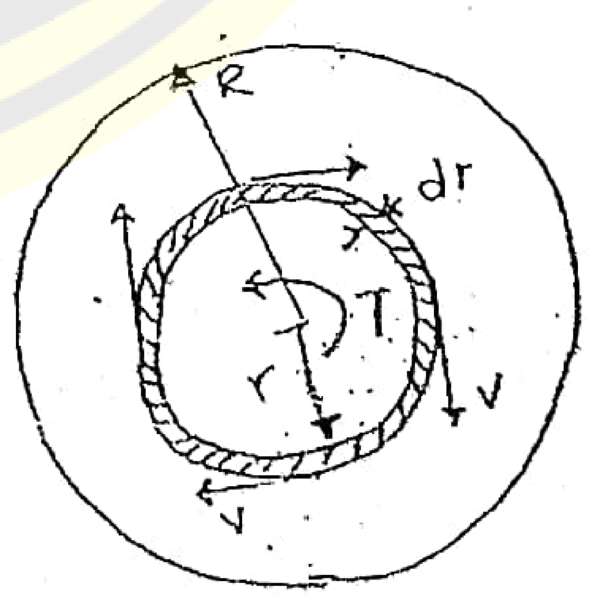
$$y = \left(\frac{\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}}\right) H.$$

Q. For the disc rotating with angular velocity ω, Find
(i) Total drag force
(ii) External torque required to maintain the ω const.

20 Marks



(i) Differential drag force.



$$dF_D = \mu\left[\frac{r\omega - 0}{h}\right] \cdot 2\pi r \cdot dr$$

$$= \frac{2\pi\mu\omega}{h} \cdot r^2 \cdot dr.$$

Total drag $= F_D$

$$= \int dF_D$$

$$= \frac{2\pi\mu\omega}{h} \int^R r^2 \, dr$$

(ii) Resistive torque

$$d\tau_D = dF_D \cdot r$$

$$= \frac{2\pi\mu\omega}{h} \cdot r^3 \cdot dr$$

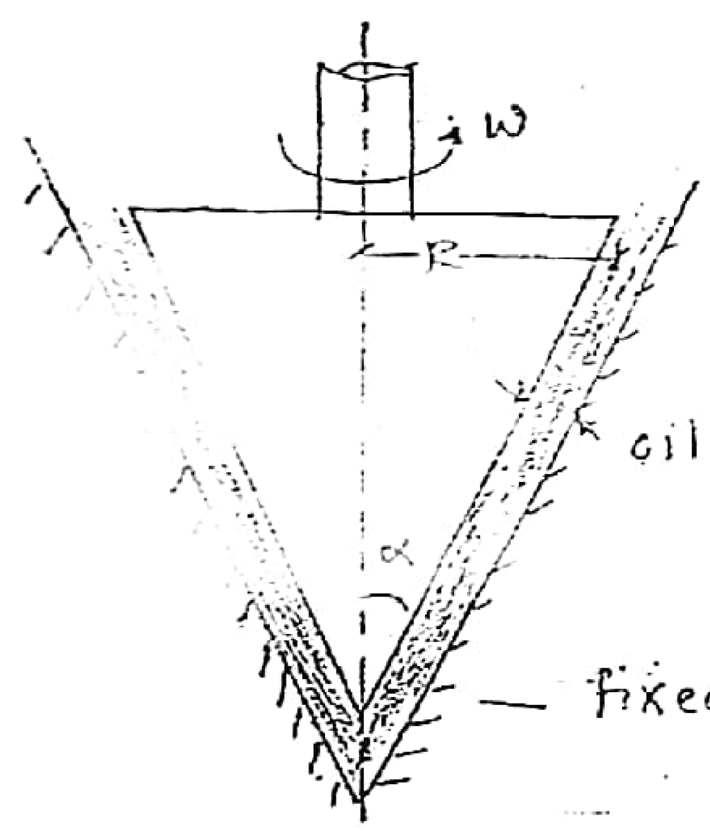
$$\tau_D = \frac{2\pi\mu\omega}{h} \int_0^R r^3 \, dr$$

$$\tau_D = \frac{\pi\mu\omega R^4}{2h}$$

Thus the external torque required to maintain the disc in angular velocity ω is of magnitude τ_D and opposite direction to that of τ_D.

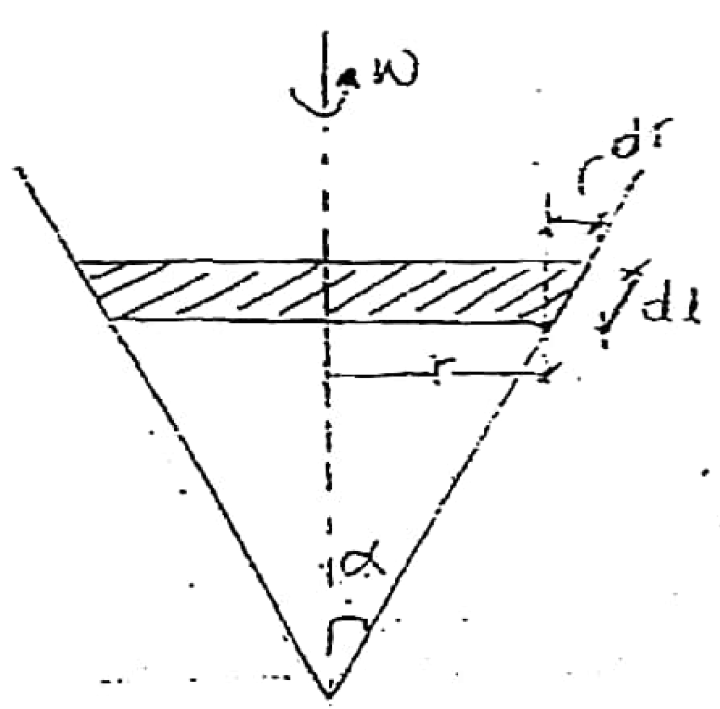
Q. Find (i) Total drag force and
(ii) External torque required to maintain the ω. for

25 Marks



Consider an element with radius r and slant height dl as shown in fig.

$$\sin\alpha = \frac{dr}{dl}$$



(i) Differential drag force:

$$dF_D = \mu \left[\frac{r\omega - 0}{h}\right] \frac{2\pi(r+r+dr)}{2} \cdot dl$$

$$= \mu \cdot \left[\frac{r\omega\pi}{h}\right] \cdot 2r \cdot dl \quad \text{— neglect } dr \cdot dl$$

$$= \frac{\mu\omega\pi}{h} \cdot 2r^2 \cdot \frac{dr}{\sin\alpha}$$

$$F_D = \int_0^R \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \cdot r^2 \cdot dr$$

$$= \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \left[\frac{R^3}{3}\right]$$

Total drag. $F_D = \dfrac{2\mu\omega\pi R^3}{3\, h \cdot \sin\alpha}$

(ii) Resistive torque

$$d\tau_D = dF_D \cdot r$$

$$= \frac{\mu\omega\pi}{h} \cdot \frac{2r^2 \cdot dr}{\sin\alpha} \cdot r$$

$$= \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \cdot r^3 \cdot dr$$

$$\tau_D = \int_0^R \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \cdot r^3 \cdot dr$$

$$= \frac{2\mu\omega\pi}{h \cdot \sin\alpha} \left[\frac{R^4}{4}\right]$$

$$\tau_D = \frac{\mu\omega\pi R^4}{2h \cdot \sin\alpha}$$

The external torque required to maintain the cone in angular velocity is of magnitude τ_D and in direction opposite to that of τ_D.

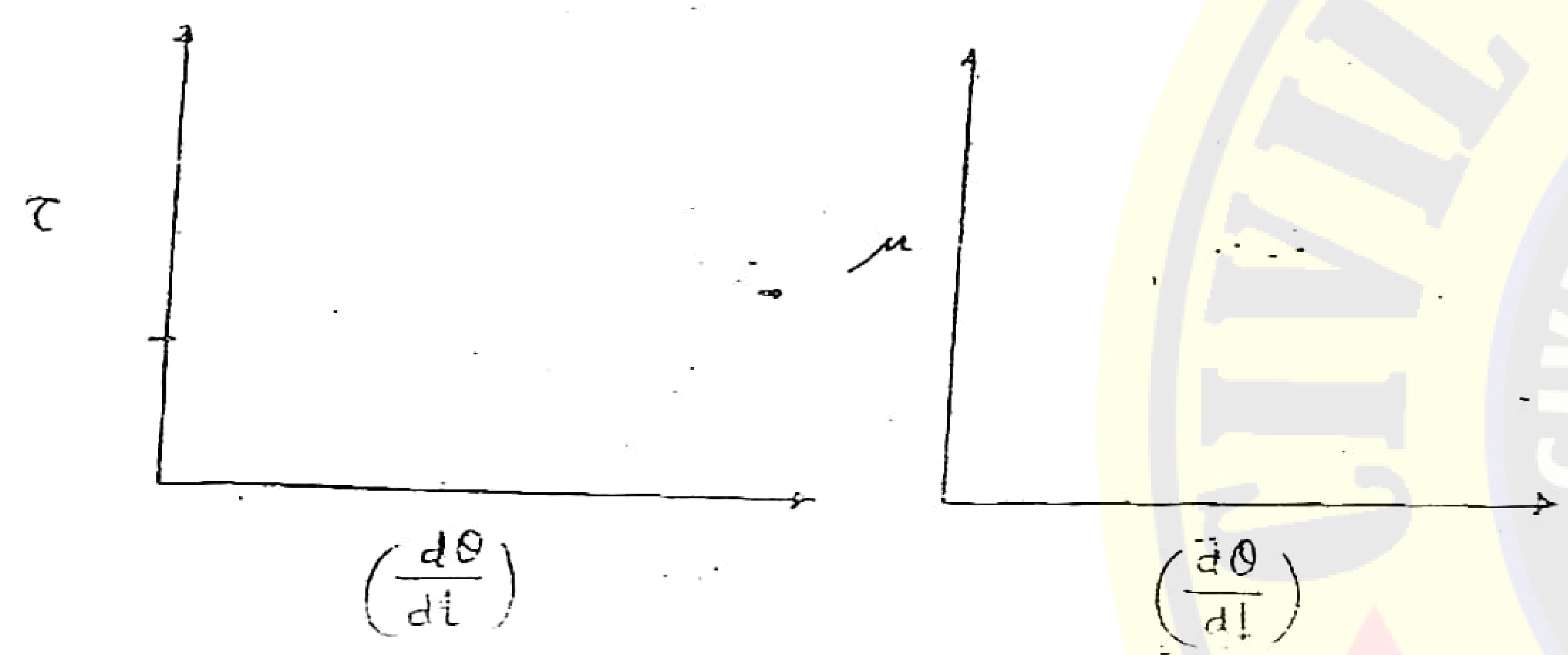
Rheology :

"It is the branch of science which deals with the studies of different types of fluid behaviour." Ratio - fluid

1. Ideal fluids :

Ideal fluids are the one which have.

- Zero viscosity (Inviscid)
- Incompressible ($\rho = 0$)
- zero surface tension effects.



(Ideal fluids [illegible] independent on flow velocity)

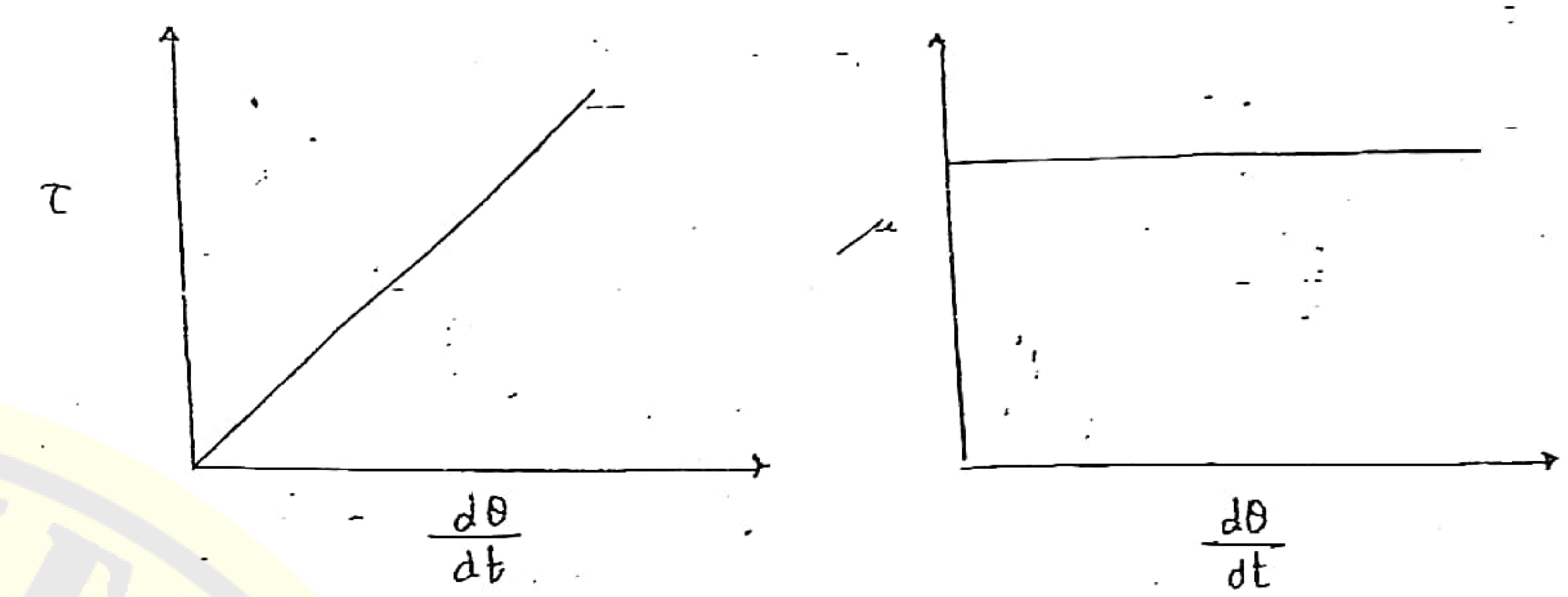
2. Newtonian fluid :

They follow the Newton's law of viscosity. They are the fluids in which viscosity does not depend upon rate of shear deformation.

$$\tau = \mu \cdot \frac{d\theta}{dt}$$

μ - is constant

e.g. Water. (spreads on surface & shear stress increases) air, petrol, oil



3. Non-Newtonian fluids :

The fluids which don't follow Newton's law of viscosity.

$$\tau = A\left(\frac{d\theta}{dt}\right)^n$$

- non linear relation
- $n > 0$ but $n \neq 1$

A is constant

The value of n may be $n > 1$ or $n < 1$.

$$\tau = A\left(\frac{d\theta}{dt}\right)^{n-1} \frac{d\theta}{dt}$$

where $A \cdot \left(\frac{d\theta}{dt}\right)^{n-1}$ Apparent viscosity ($\mu_{app.}$)

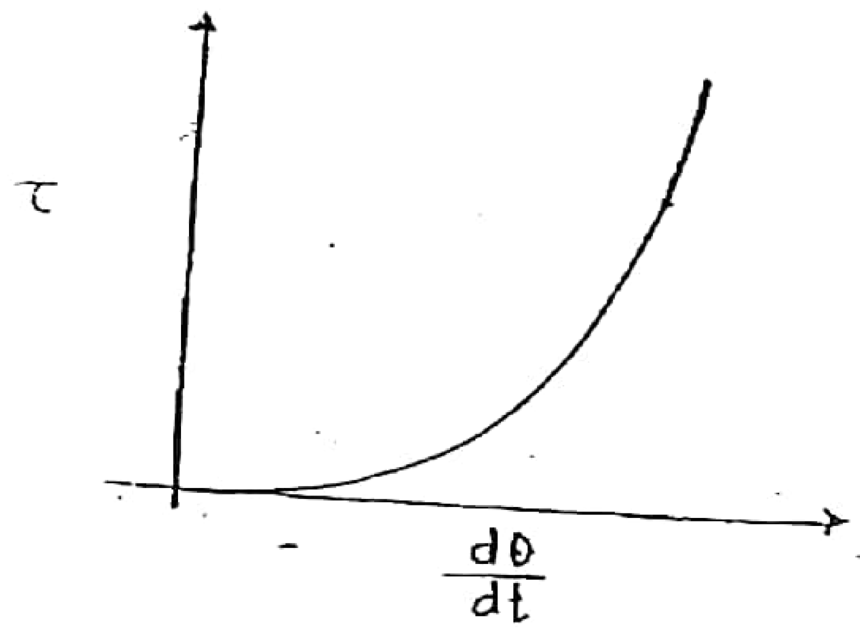
The apparant viscosity of Non-newtonian fluids depends on rate of shear deformation.

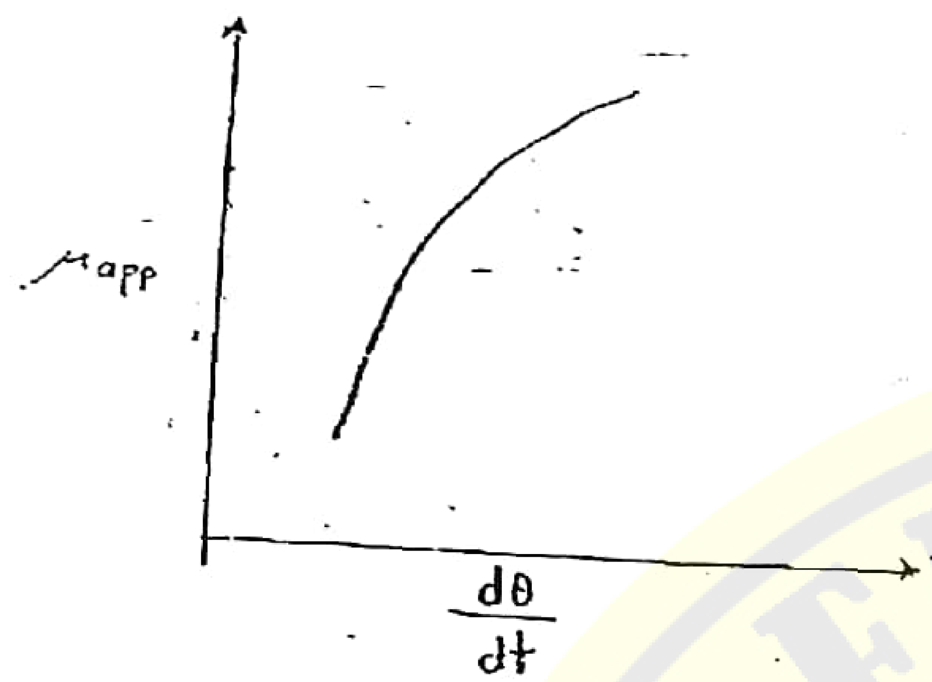
It may be of two types

(i) Dilatent fluids.
($n > 1$)

$\mu_{app.}$ increases with rate of shear deformation.

They are also called 'Shear thickning fluid'.



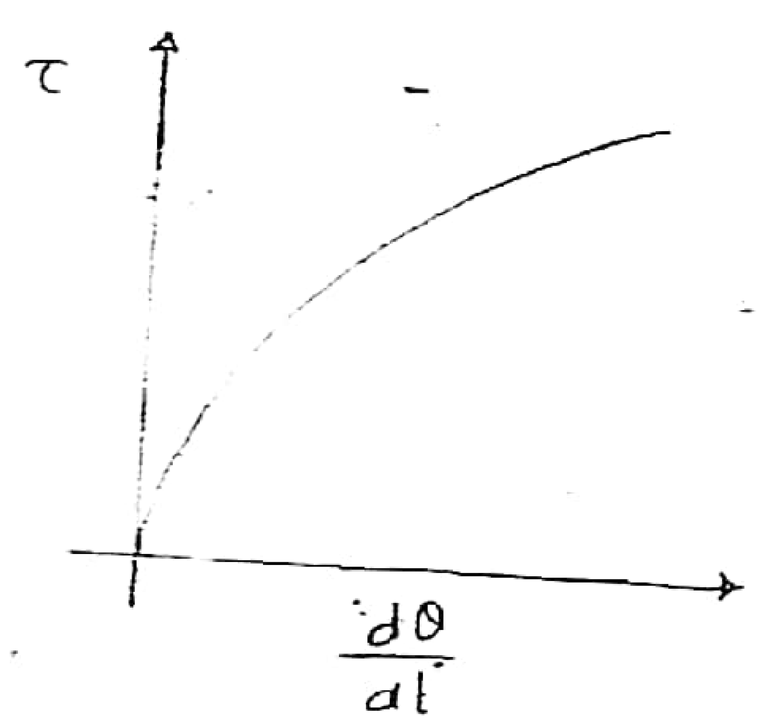


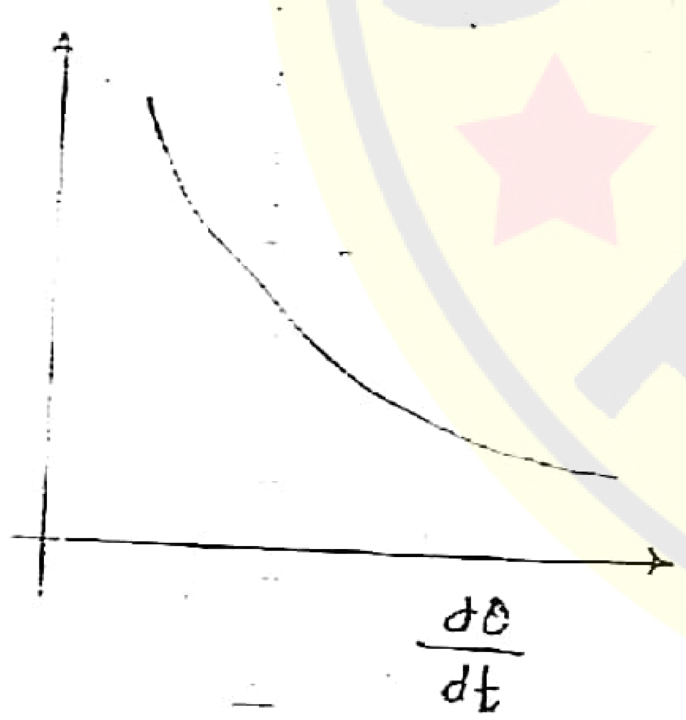
e.g. saturated sol^n of sugar in water, ~~rice~~ starch, sewage honey.

(ii) Pseudo plastic fluid:

$(n < 1)$

The shear deformation rate decreases the apparent viscosity. They are also called Shear Thinning fluid.



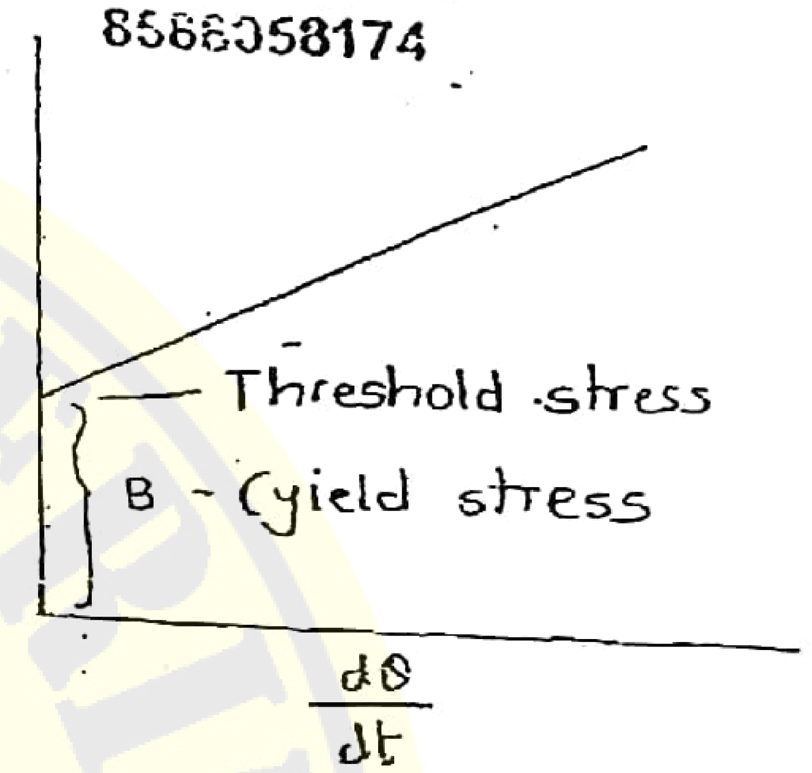


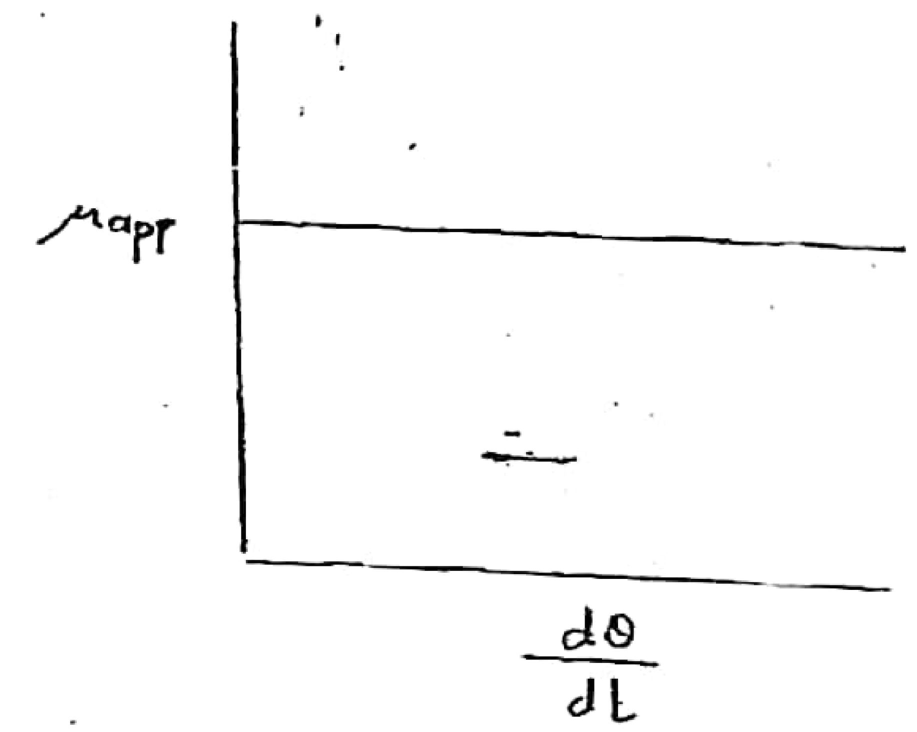
e.g. Milk, blood.

4. Ideal Bingham plastic fluid:

$$\tau = A \cdot \left(\frac{d\theta}{dt}\right) + B$$

A, B are positive constants.





e.g. Tooth paste, Hair gel, Cosmetic creams.

We need to overcome yield stress to make these fluids flow by applying shear stress.

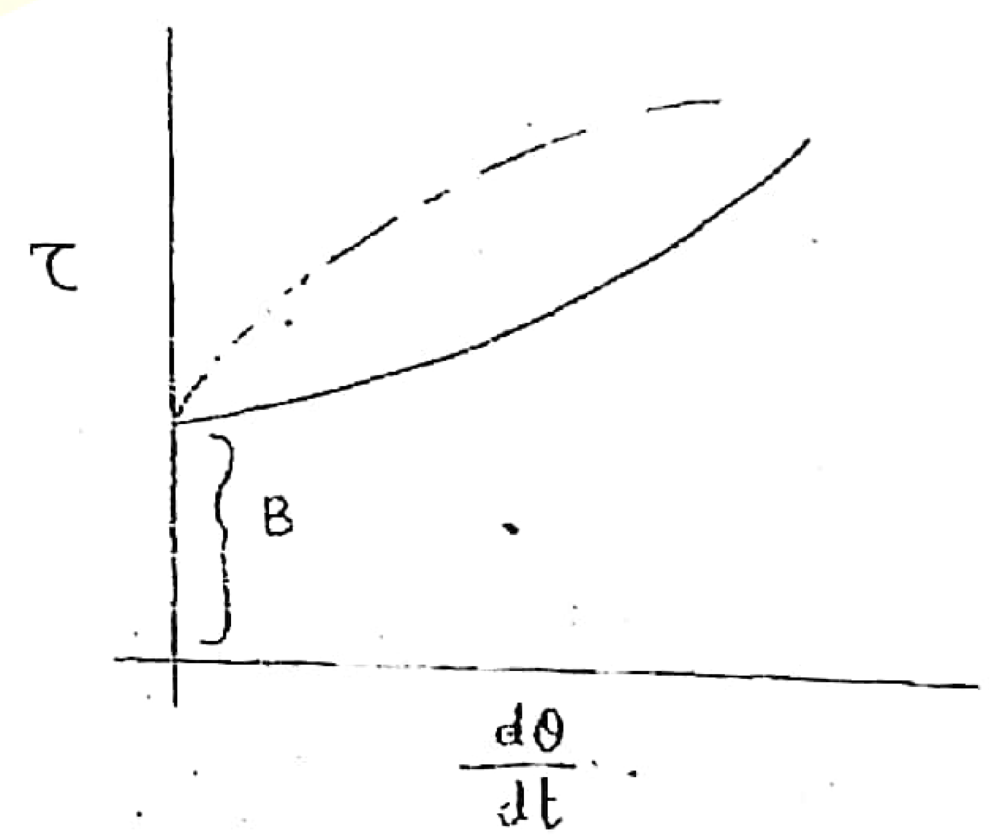
5. Thixotropic fluid.

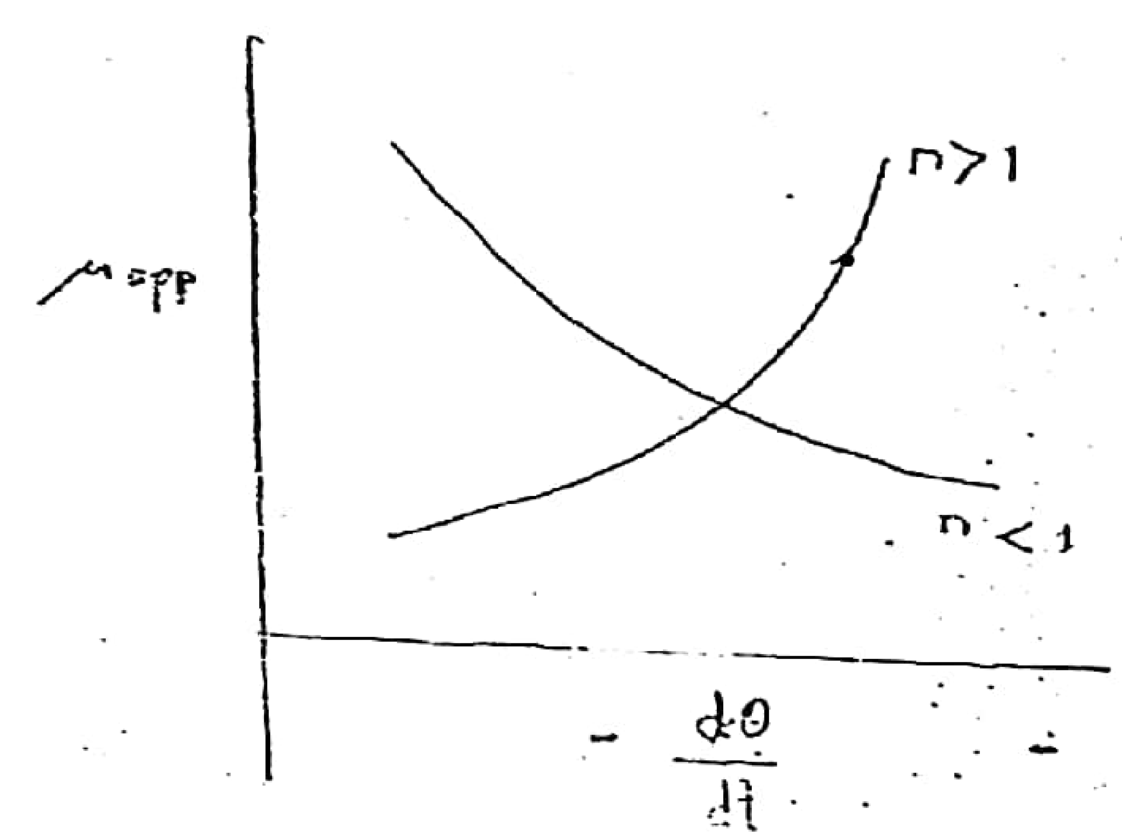
$$\tau = A \cdot \left(\frac{d\theta}{dt}\right)^n + B$$

A, B > 0

$n \neq 1$

n may b $n > 1$, $n < 1$





The viscosity of thixotropic fluids also depends upon the time.

If μ increases with time, the fluids are called grade 1

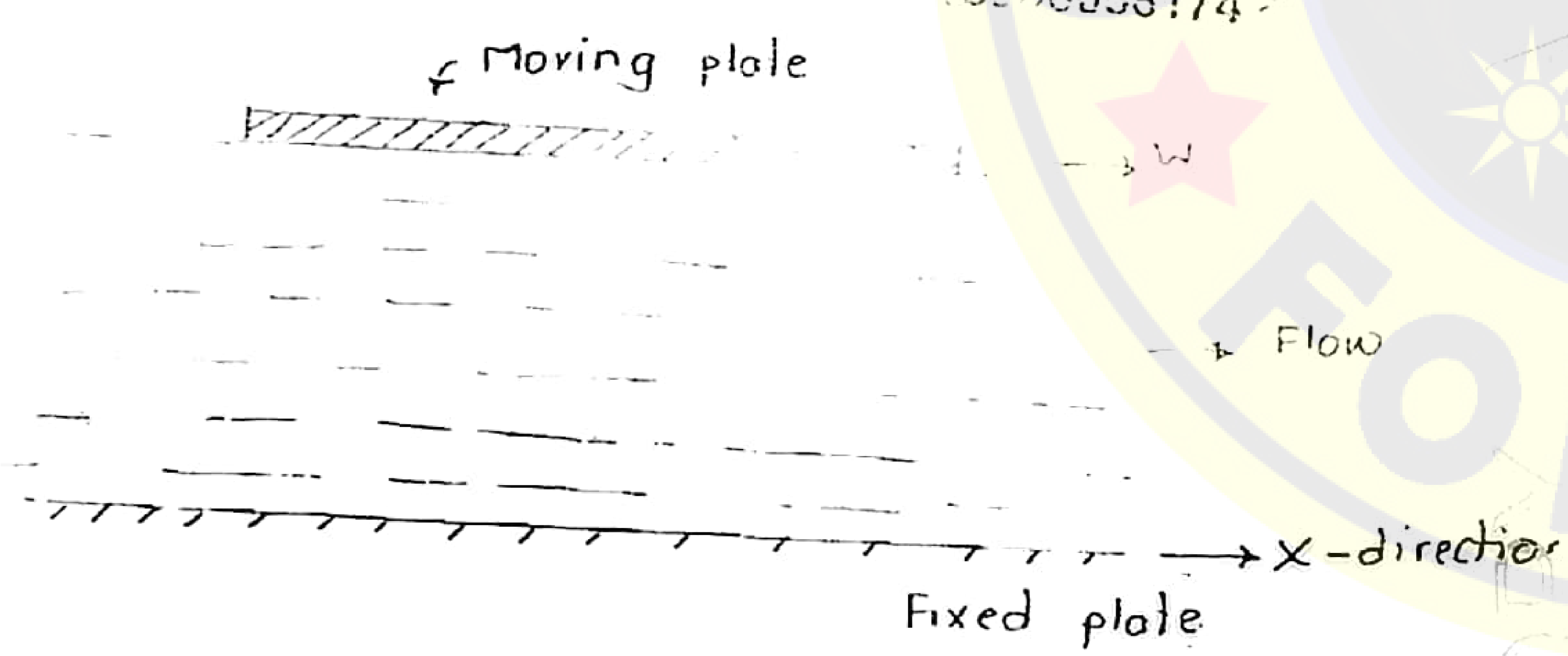
If μ decreases with time, the fluids are called grade 2. (also called as Rheopectic fluids)

e.g. Printer's ink, paints, pediatric drugs, chemotropic drugs, glucose in powder form.

Visco-elastic fluids:

This is a fluid which shows the property of the elasticity upto certain limits. Beyond that they are continuously deformed under shear force.

e.g melted rubber



shear stress on upper plate will be in $\rightarrow$ -ve X-direction

shear stress on contacting layer with upper plate will be $\rightarrow$ +ve X-direction

shear stress on contacting layer with lower plate will be $\rightarrow$ -ve X

shear stress on lower plate will be in $\rightarrow$ +ve X-direction

Note:

At a constant temperature, if pressure is increased,

$\mu_{liq} \rightarrow$ doesn't change

$\nu_{liq} \rightarrow$ doesn't change.

$\mu_{gas} \rightarrow \because C_{rms} = \sqrt{\frac{3RT}{M}}$ T-constant

$\therefore$ No change in μ_{gas}

$\because \rho \propto P$ $\therefore \nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$

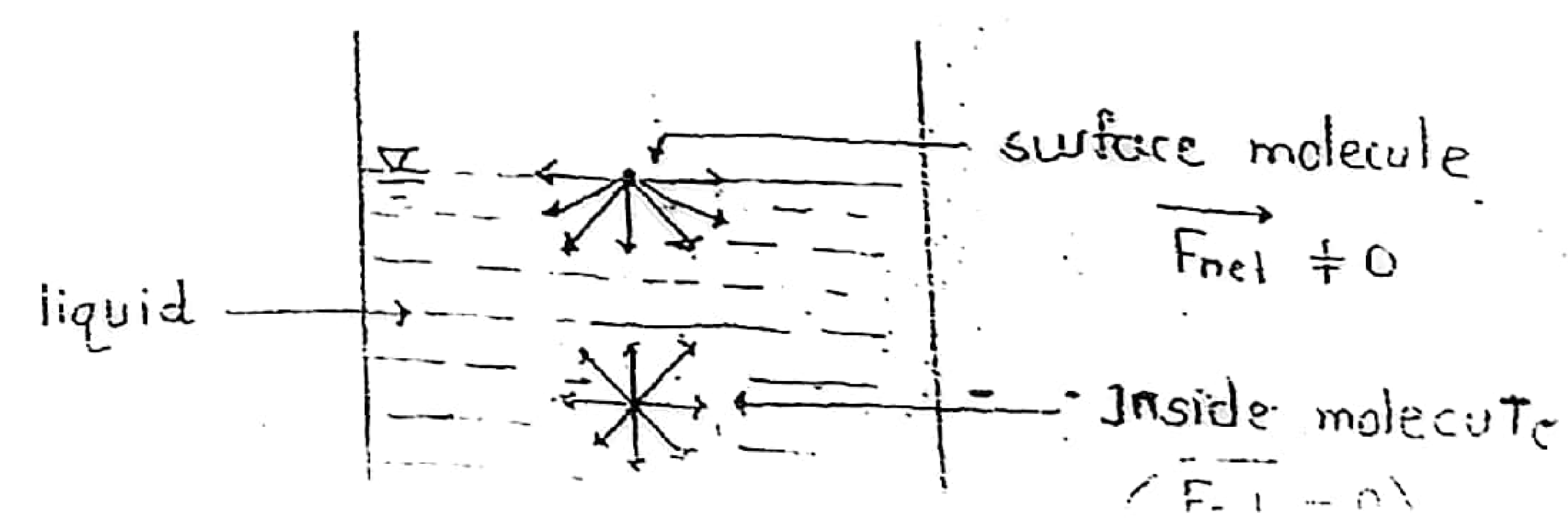
ν_{gas} decreases with increase in pressure.

Surface tension:

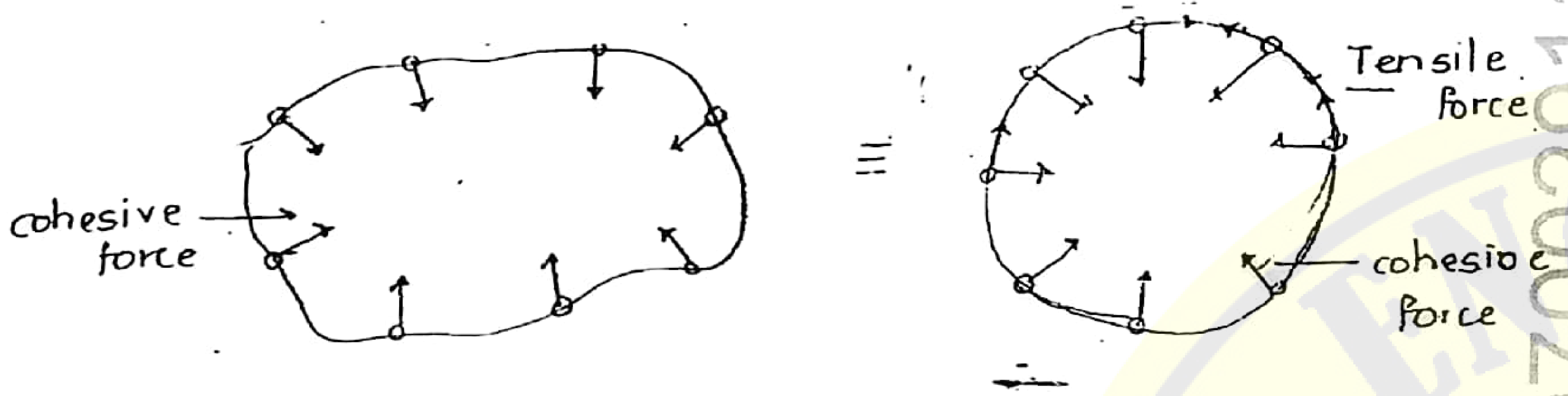
"Every fluid is having property of minimising its surface-area upto its maximum extent. Such a property of fluid is known as Surface tension". The only reason of this property is the cohesion i.e. cohesive force between the molecules of fluid.

Therefore, it is mainly the property [illegible] because the cohesion in the gases is almost nill.

For the same volume, sphere has the minimum surface area ($4\pi r^2$) amongst the all geometric shapes.



The surface molecule is pulled by the other molecules in inward direction.



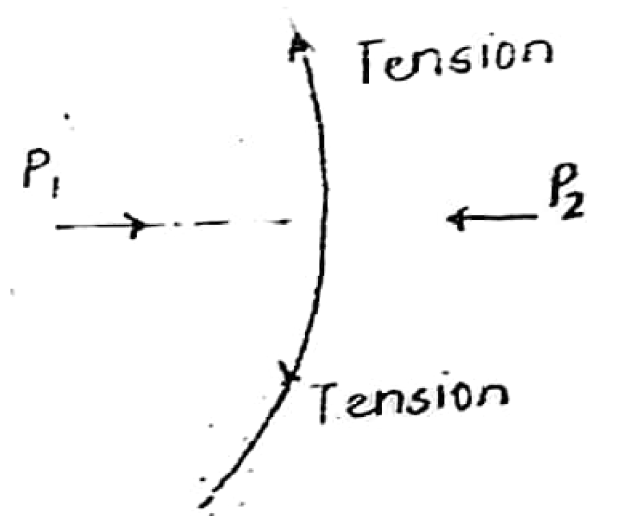
Mathematically surface tension is defined as the force per unit length of free surface.

$$\sigma = \frac{F}{l} \quad N/m$$

where, F is the tensile force developed at the surface due to cohesion.

i.e. cohesive force is cause

surface tension is the effect

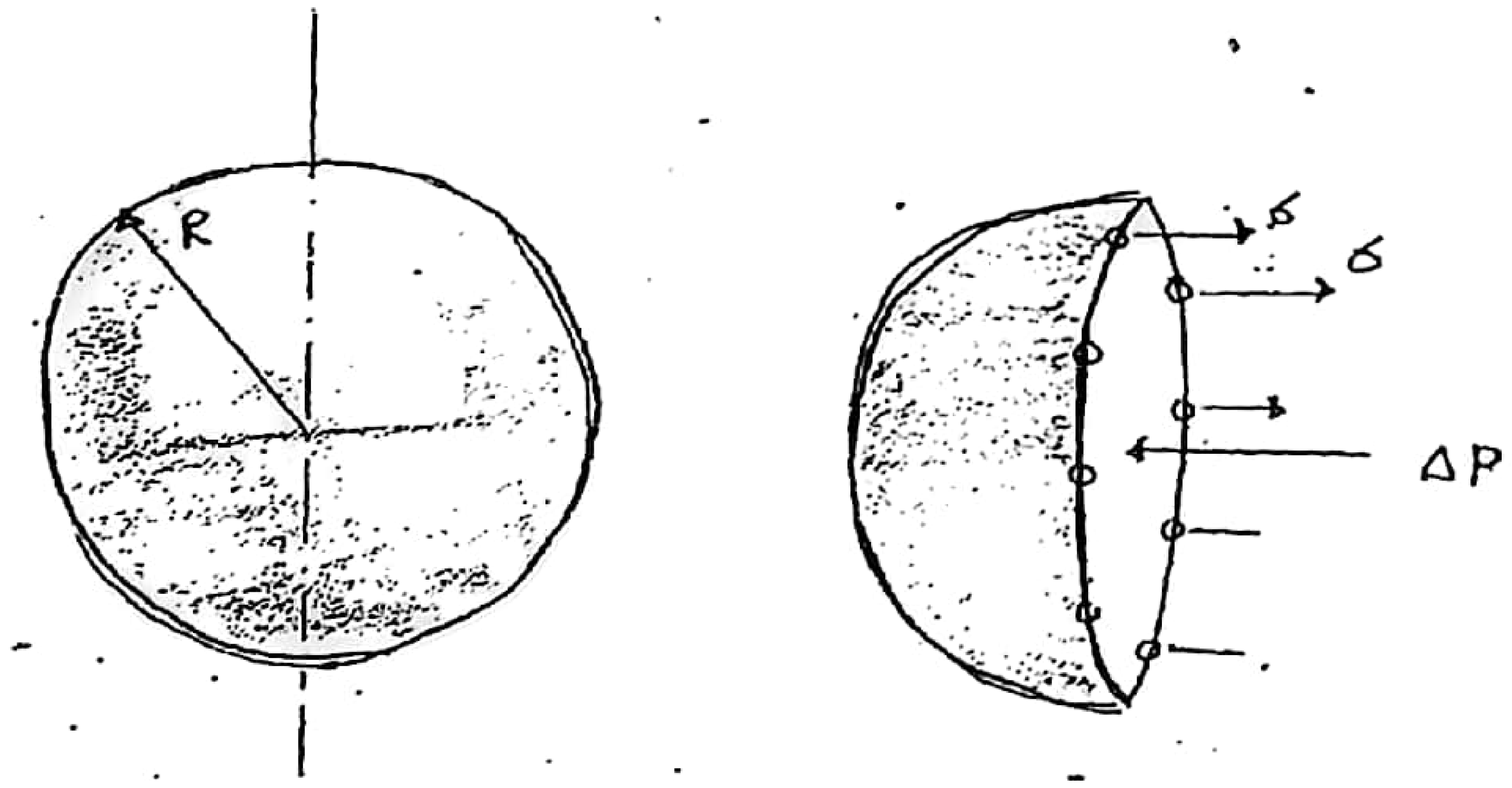


$P_1 > P_2$

Excess pressure on concave side is,

$$(P_1 - P_2) = \Delta P.$$

Drop formation :



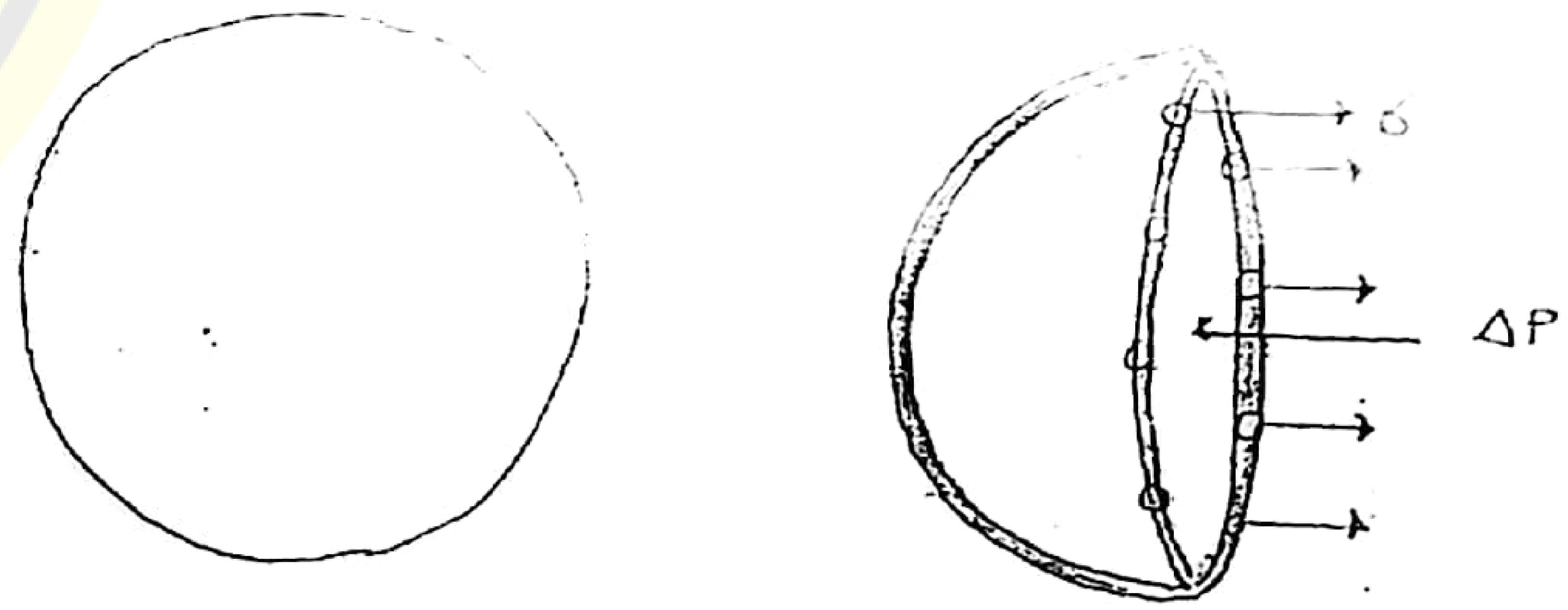
Surface tension force (σ) acting on $2\pi R$.

Excess pressure (ΔP) acting on inner concave.

$$\Delta P \cdot \pi \cdot R^2 = \sigma \cdot 2\pi R.$$

$$\Delta P = \frac{2\sigma}{R}.$$

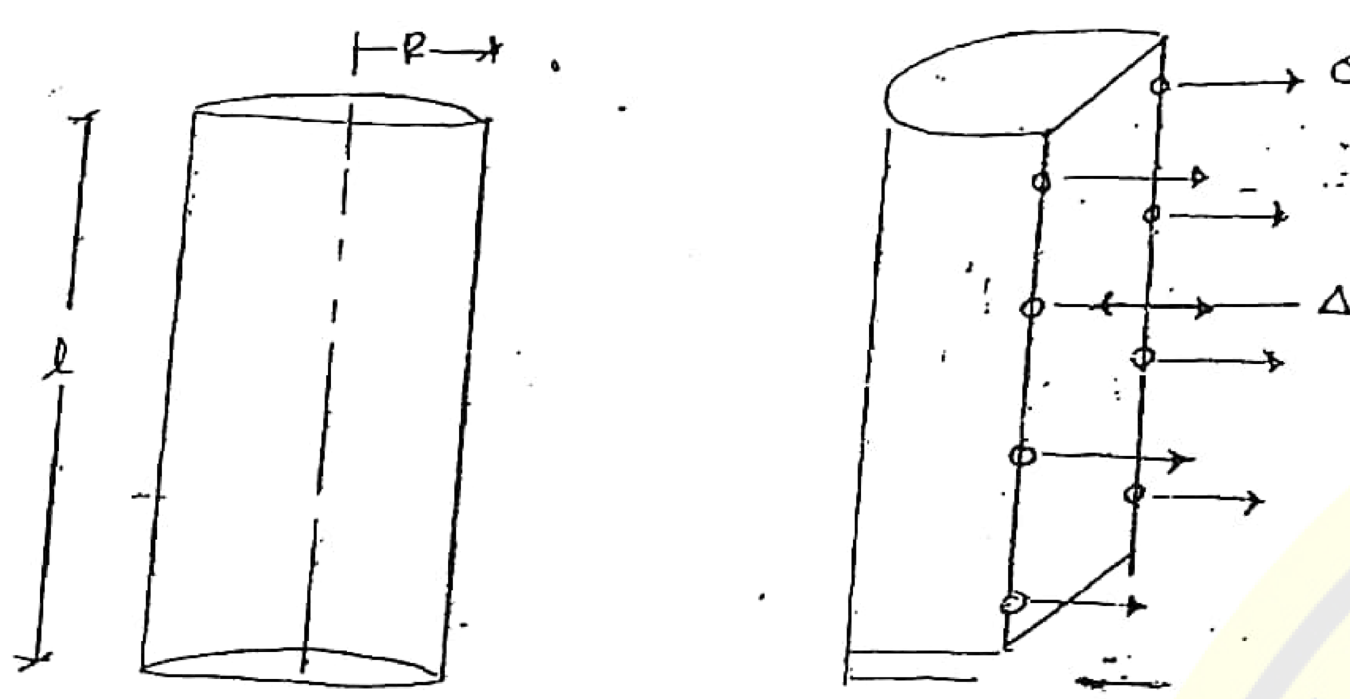
Bubble formation :



$$\Delta P \cdot \pi R^2 = \sigma \cdot 4\pi R$$

$$\Delta P = \frac{4\sigma}{R}.$$

Jet formation:



$$\Delta P \cdot (2R \cdot l) = \sigma \cdot 2l$$

$$\Delta P = \frac{\sigma}{R}$$

The formation of jet is easy as it requires least excess pressure. So the jet will form first when we apply excess pressure.

The working fluid filled inside — called as drop.

e.g. Water drop has water filled inside.

air bubble has air inside as working fluid, thus air bubble is a Drop.

Wednesday
23rd October 2013

Concept of surface energy:

The work done in creating the surface area of the fluid film is stored in the form of potential energy and it is known as Surface energy.

The higher potential energy state always tries to get converted to lowest potential energy state. i.e. The lowest potential energy state is highly stable state.

The body always tries to acquire low potential

Work = Force × displacement.

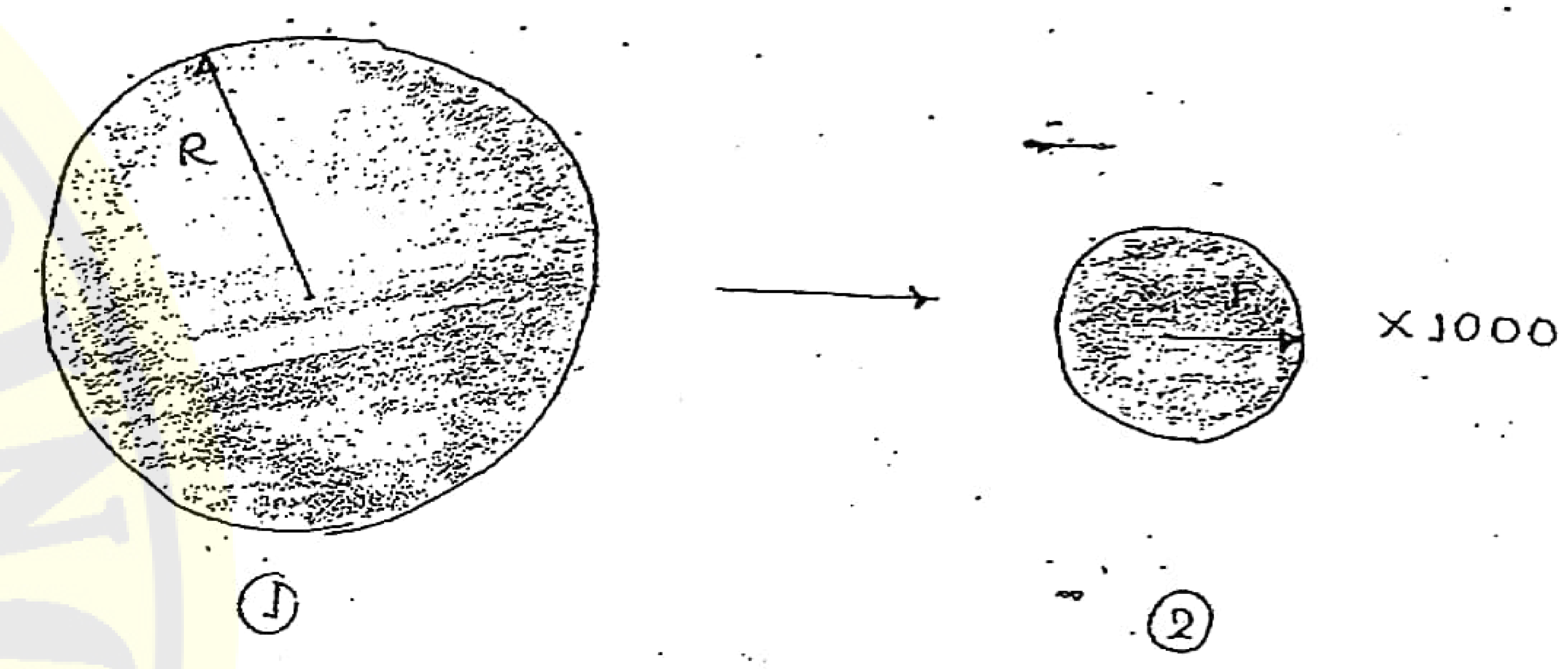
for length l, to strech the material (form film) by x.

Work $= (\sigma \cdot l) \cdot x$ $\quad$ $\sigma \cdot l$ – Force i.e. surface tension

$= \sigma\,(lx)$

$E = \sigma \cdot A$

e.g.



$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$$r = \frac{R}{10}$$

For state ①

$E_1 = \sigma_1 (4\pi R^2)$ – lower energy state

For state ②

$E_2 = \sigma (4\pi r^2) \times 1000$

$= \sigma \left(4\pi \times \frac{R^2}{100}\right) \times 1000$

$= 10\,\sigma (4\pi R^2)$ – higher energy state

$\Delta E = E_2 - E_1$

$= 36\,\sigma \pi R^2$

Wetting and non wetting fluids:

It is a mutual property of liquid and surface. It depends on cohesion and adhesion both.

If adhesion >>>> cohesion.

Liquid is wetting the surface.

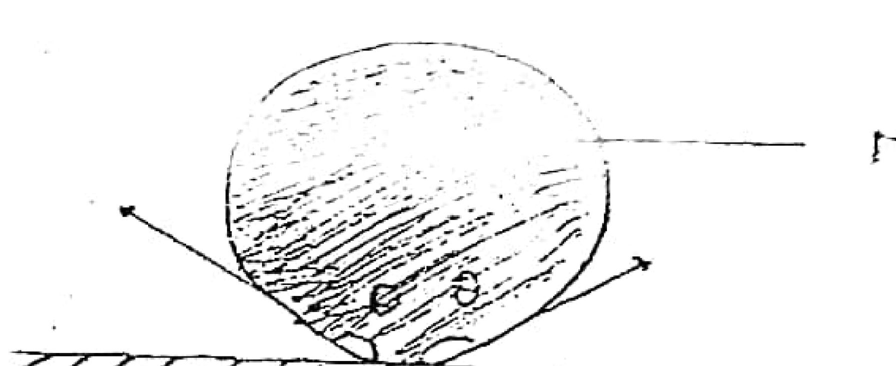
e.g. Water on the glass (angle of contact $\theta < \pi/2$)

For pure water, angle of contact is 0° (wets surface completely)

If adhesion <<<< cohesion.

Liquid is non wetting the surface

e.g. Mercury on the glass ($\theta > \pi/2$)

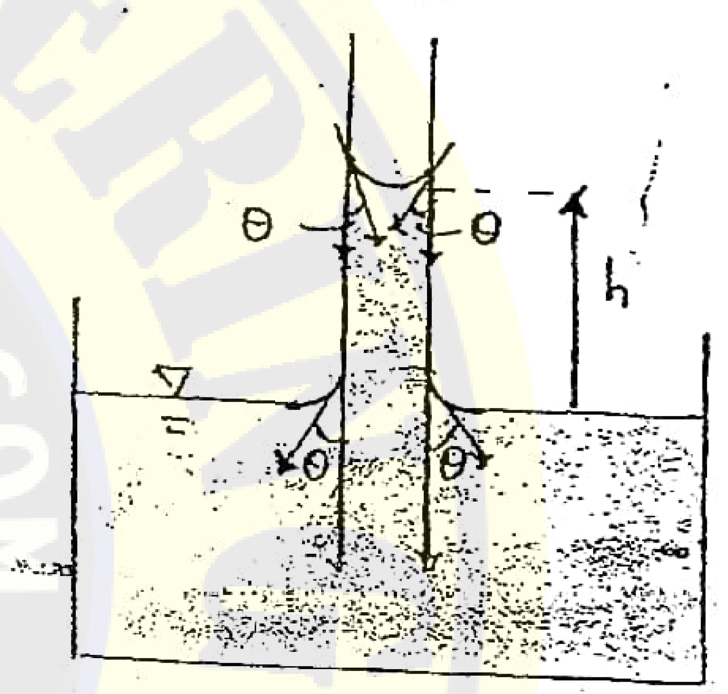


Mercury actually touches the glass surface at a point when taken in small quantity. but when large drop is placed on glass surface, contact area is more due to the weight of drop itself.

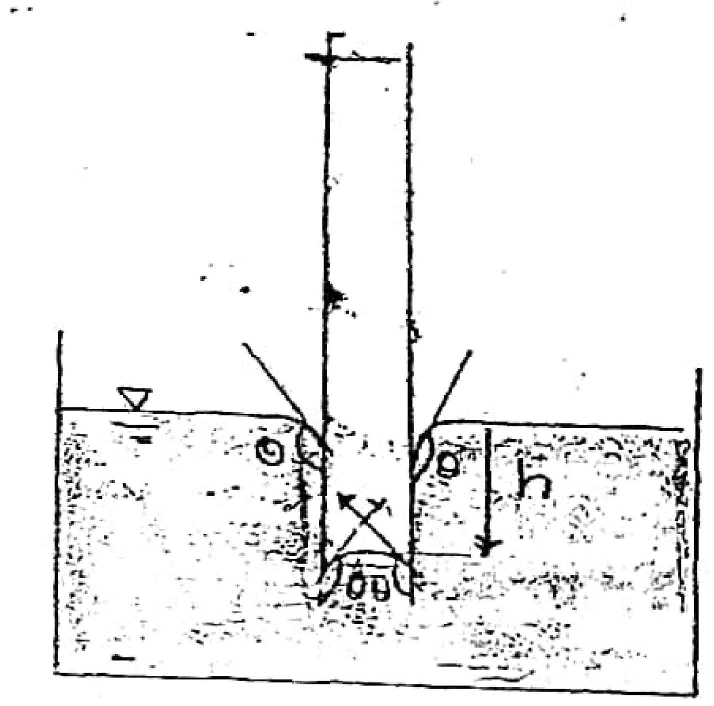
Angle of contact also depends upon the type of contact surface.

Capillarity:

When a tube of very fine diameter is submerged or emersed inside a fluid then there may be the rise or fall of liquid level inside the tube, depending upon the wetting and non-wetting nature of liquid, with tube surface. This rise or fall of the liquid level in a tube of fine diameter is a phenomenane known as Capillarity. and this tube of fine diameter is known as Capillary tube.

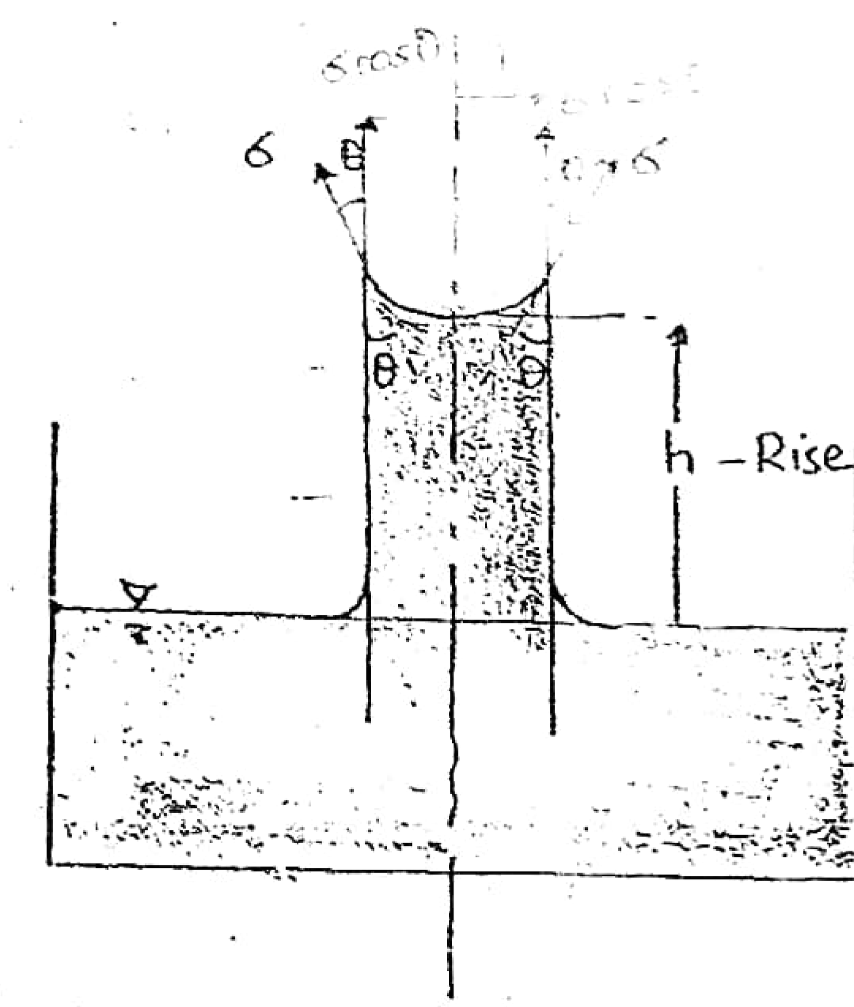


Wetting ($\theta < \pi/2$)



Non-wetting ($\theta > \pi/2$)

Rise or fall

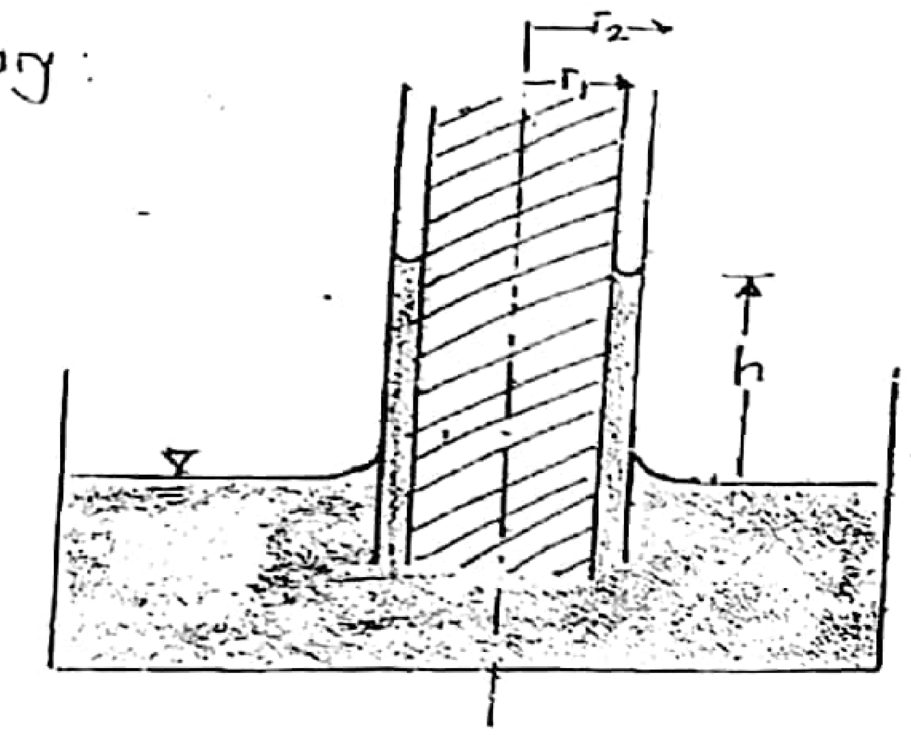


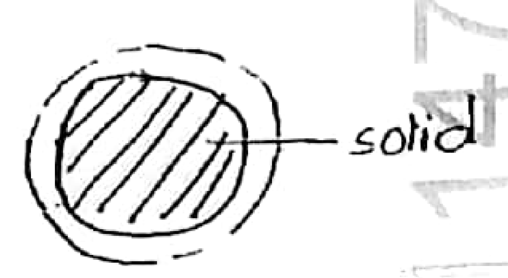
liquid (angle of contact - θ)

$$\sigma \cos\theta \cdot 2\pi r = (\pi r^2 \cdot h) \cdot \rho g$$

$$h = 2\sigma \cos\theta$$

Annular capillary :



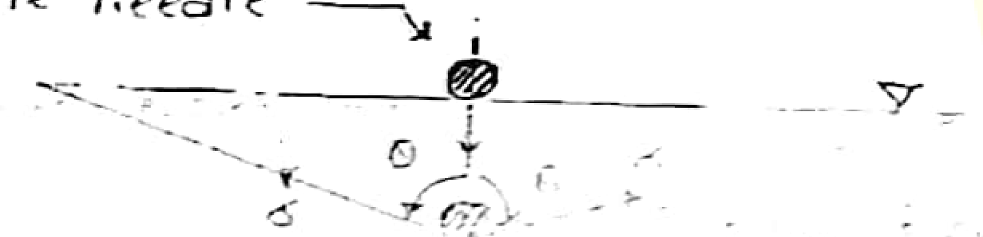


$$\sigma \cos\theta \cdot (2\pi r_1 + 2\pi r_2) = \pi (r_2^2 - r_1^2) \cdot h \cdot \rho \cdot g$$

$$2\pi \cdot \sigma \cos\theta \, (r_1 + r_2) = \pi (r_1 + r_2)(r_2 - r_1) \cdot h \cdot \rho \cdot g$$

$$h = \frac{2\sigma\cos\theta}{(r_2 - r_1) \cdot \rho g}$$

Stainless steel needle floating on surface of water.



[illegible] surface.

Needle sinks down due [illegible] weight.

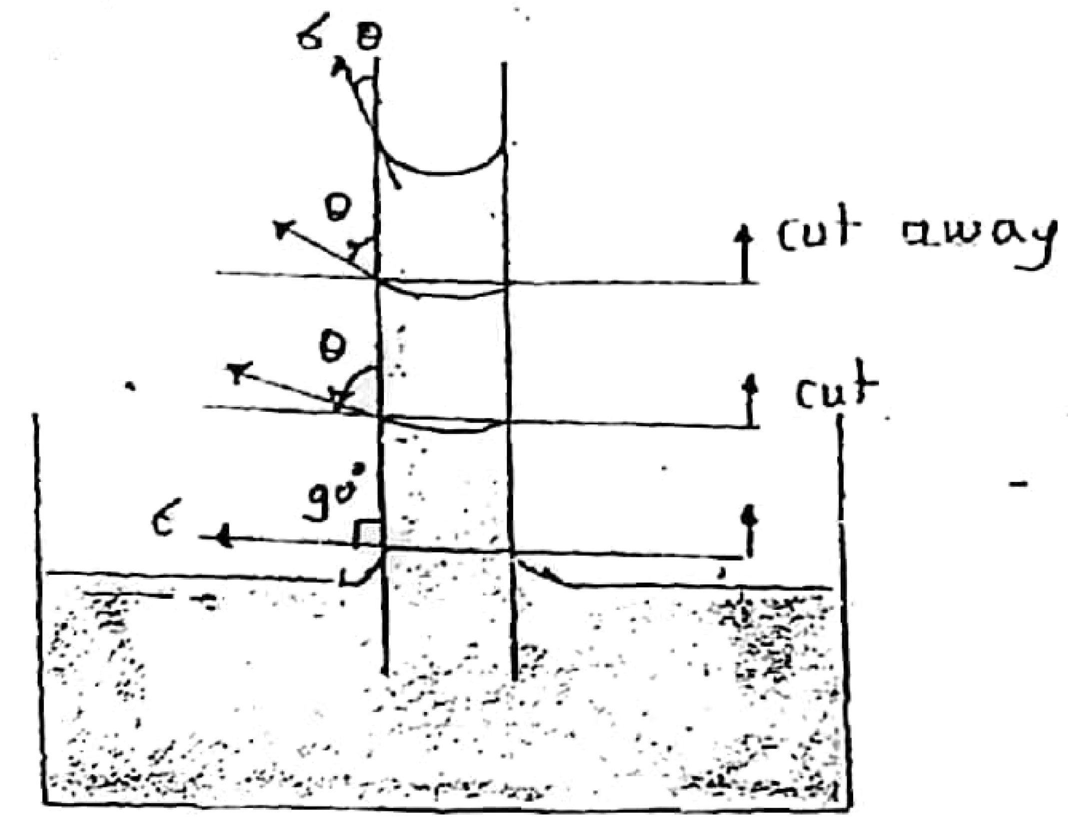
$$(2\sigma\cos\theta) \cdot l = mg$$

l - length of needle
m - mass of needle

$$(2\sigma\cos\theta) \cdot l = (\pi r^2 l) \cdot \rho \cdot g$$

$$2\sigma\cos\theta = \pi r^2 \rho g$$

r - radius of needle.



If for certain fluid (angle of contact θ) the rise in capillary tube is h and if the length of tube is less than h then.

(i) Cocavity of the free surface of fluid (liquid) inside capillarity will decrease

(ii) Angle of contact (θ) will increase

(iii) But the water will not spill out of capillary.

If the length of capillary is further reduced to the liquid [illegible] container (angle of [illegible] will become 90°) and [illegible] meniscus will become horizontal.

Pressure and its measurement :

Pressure :-

The existance of molecules in a system is existance of pressure.

Mathematically, intensity of pressure is defined as external normal force per unit area.

$$P = \frac{F}{A} \rightarrow \text{External normal force (thrust)}$$

(gives pressing effect)

Pressure is scalar quantity (direction - intensity is same in all directions) but pressure force is a vector.

Units of pressure :

(i) $1 \text{ Pa} = 1 \text{ N/m}^2$ S.I. unit

(ii) $1 \frac{kg \cdot f}{cm^2} = \frac{9.81 \text{ N}}{10^{-4} \text{ m}^2} = 9.81 \times 10^4 \text{ N/m}^2$

(iii) $1 \text{ bar} = 10^5 \text{ Pa}$

(iv) $1 \text{ atm} = 101.325 \text{ Pa}$

(v) FPS system.

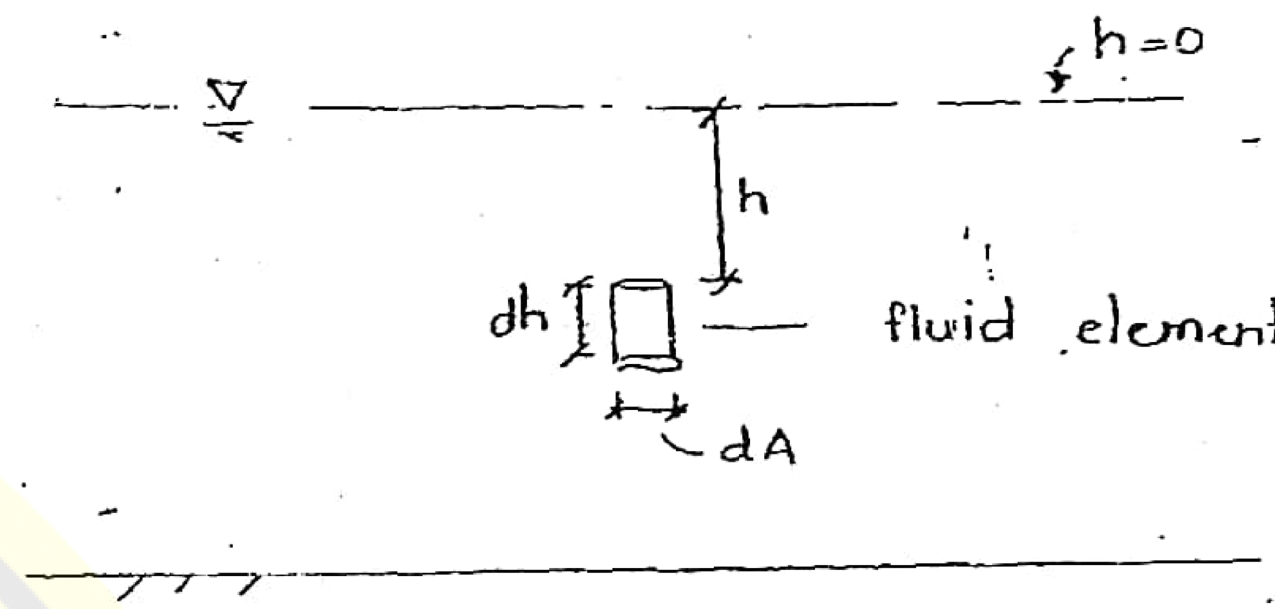
$1 \frac{lb \cdot f}{inch^2} = 1 \text{ P.S.I}$ (pound per square inch)

$= \frac{0.453 \text{ kg} \cdot f}{(2.54 \text{ cm})^2} = \frac{0.453 \times 9.81}{(2.54)^2 \times (10^{-2})^2}$

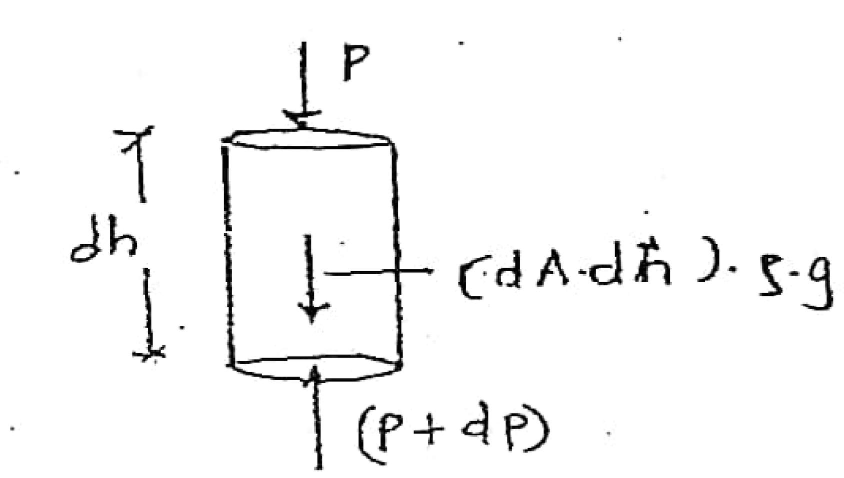
$= 6888.1 \text{ Pa}$

$1 \text{ atm} = 14.7 \text{ PSI}$

Pressure at a point in static fluid
(Hydrostatic pressure)



Fluid element system :



As it is static fluid.

$(P + dP) \cdot dA - P \cdot dA - (dA \cdot dh) \cdot \rho g = 0$

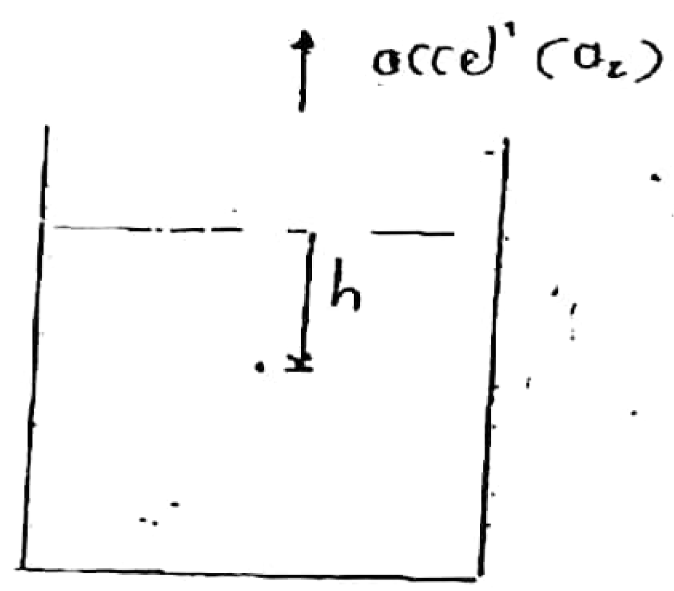
$\frac{dP}{dh} = \rho g$ (> 0 means the pressure increasing with height)

$\int_0^P dP = \int_0^h \rho \cdot g \cdot dh$

P = 0 → at h = 0 because we are considering only fluid pressure.

$P = \rho g h.$

If the container is moving upward with acceleration a_z



$$(P+dP)\cdot dA - P\cdot dA - (dA\cdot dh)\cdot \rho g = (dA\cdot dh)\cdot \rho\cdot a_z$$

$$\frac{dP}{dh} = \rho(g+a_z)$$

$$P = \rho(g+a_z)\cdot h$$

If container is moving downward with acceleration a_z

$$P = \rho(g-a_z)\cdot h$$

For the free fall of container.

$$a_z = g$$

$$P = \rho(g-g)\cdot h$$

$$= 0 \quad \text{(weighless condition)}$$

6th Unit of pressure:

Pressure can be represented by height of fluid column (as the pressure at bottom of fluid is $\rho g h$)

$$1 \text{ atm} = 103.325 \text{ Pa}$$

In terms of water column.

$$101.325 = 1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times h_{water}$$

$$h_{water} = 10.3 \text{ m}$$

In terms of Mercury

$$101325 = 13600 \times g \times h_{Hg}$$

$$h_{Hg} = 760 \text{ mm}$$

$$1 \text{ atm} = 10.3 \text{ m water} \approx 760 \text{ mm Hg}$$

Different types of pressures:

1. Atmospheric pressure:

The pressure exerted by environmental mass is known as atmospheric pressure.

2. Absolute pressure:

It is the total pressure of the system measured from zero level. This pressure is also known as net pressure

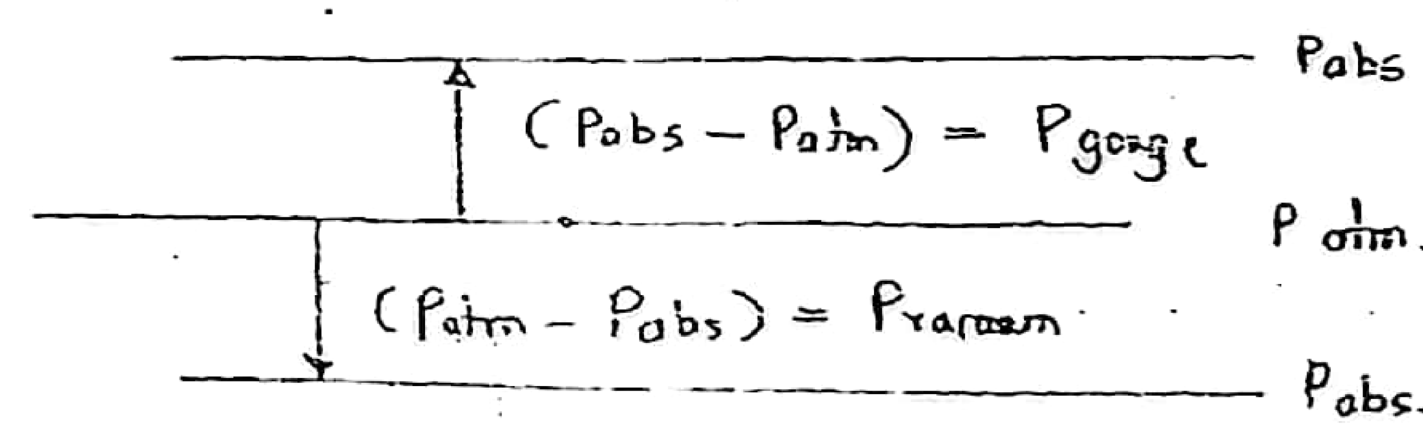
3. Gauge pressure:

It is the pressure of the system measured above atmospheric pressure values.

4. Vacuum pressure:

It is a pressure of the system measured below the atmospheric pressure values.

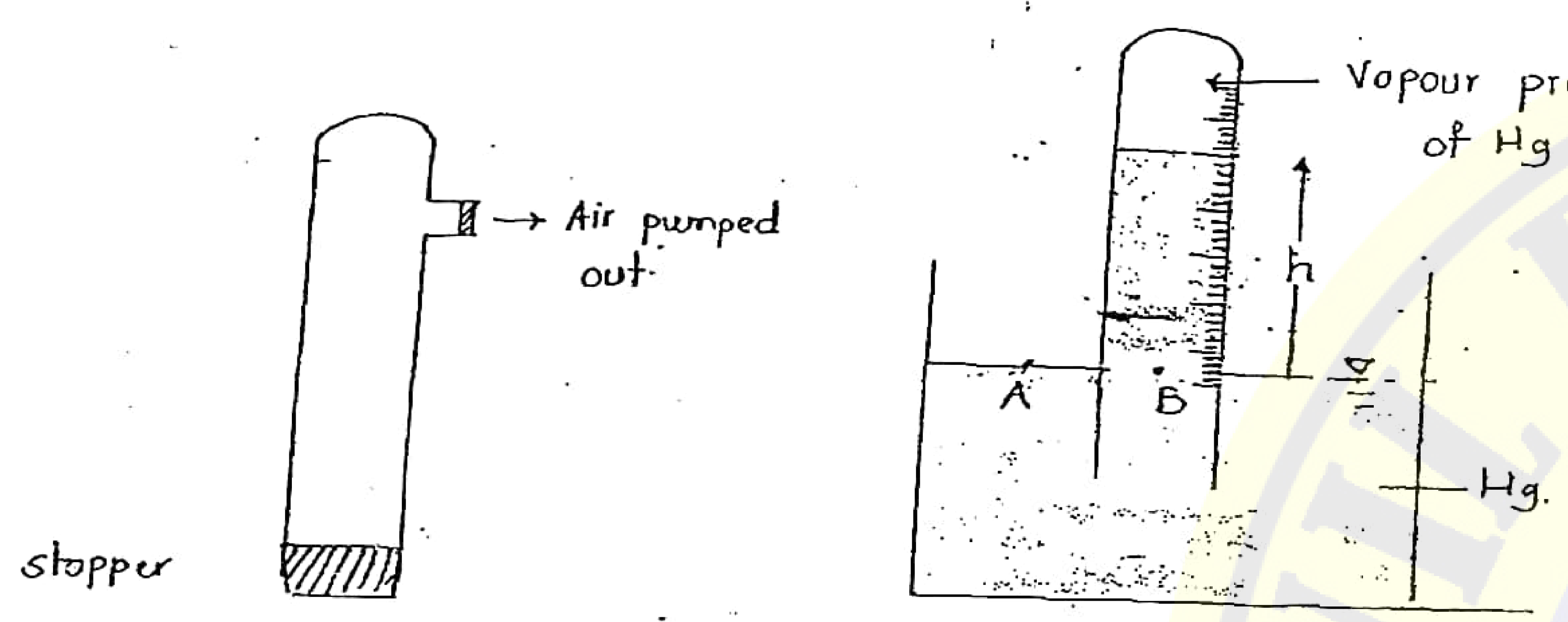
Vaccum pressures are negative gauge pressures and gauge pressures are negative vacuum pressures



Conventional pressure measurement devices:

(1) Barometer:

It is a device which is basically used to measure local atmospheric pressure. This device is made by Torri. Cell.



Vapoure pressure is pressure due to the dissolved gases in the liquid.

Here the Mercury rise is not due to capillary action but the pressure difference at the levels of Mercury at A & B when the stopper is just removed.

$$P_A = P_B \quad \text{(same liquid same height)}$$

$$P_{atm} = P_{vapour (Hg)} + \rho_{Hg} \cdot g \cdot h$$

Vapour pressure of Hg is very less, hence can be neglected while vapour pressure of water is $\frac{1}{3}$ rd of atm. pressure which cannot be neglected.

$$P_{atm} = \rho_{Hg} \cdot g \cdot h.$$

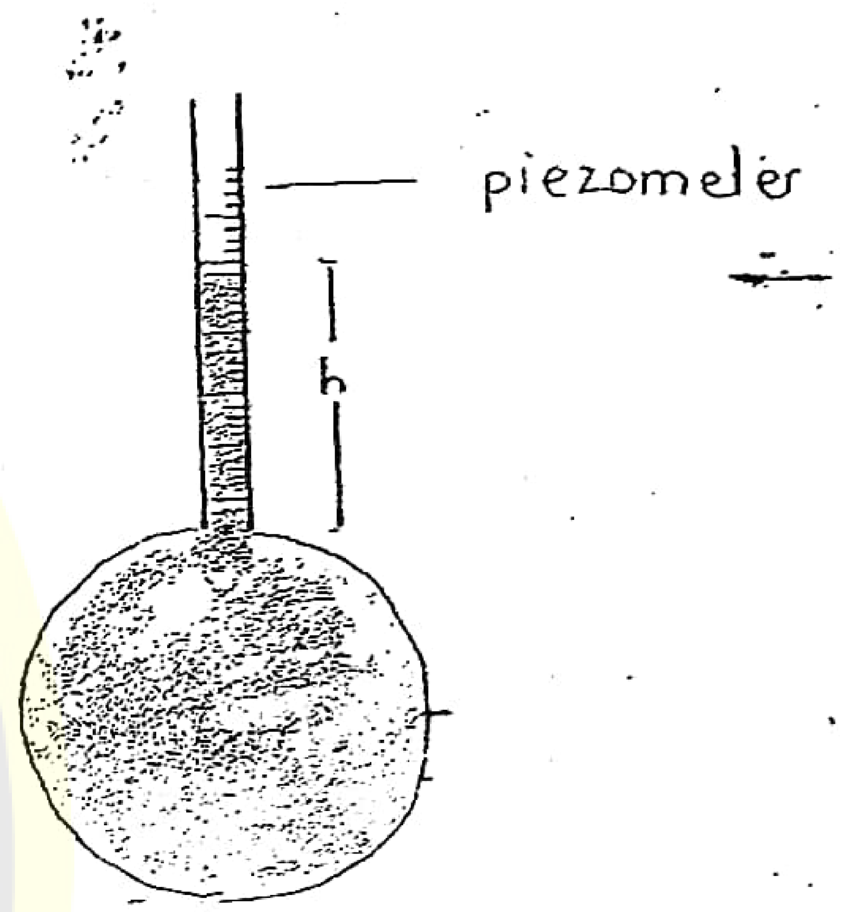
Hg is used in pressure measuring devices because.

(i) It has very high density (less column height)
(ii) It has very low vapour pressure (can be

(2) Piezometer.

It is a straight tube used for the pressure measurement

It measures only moderate flow, medium
It measure gauge (+ve) pressures
It measure only liquid pressure



(3) Manometers:

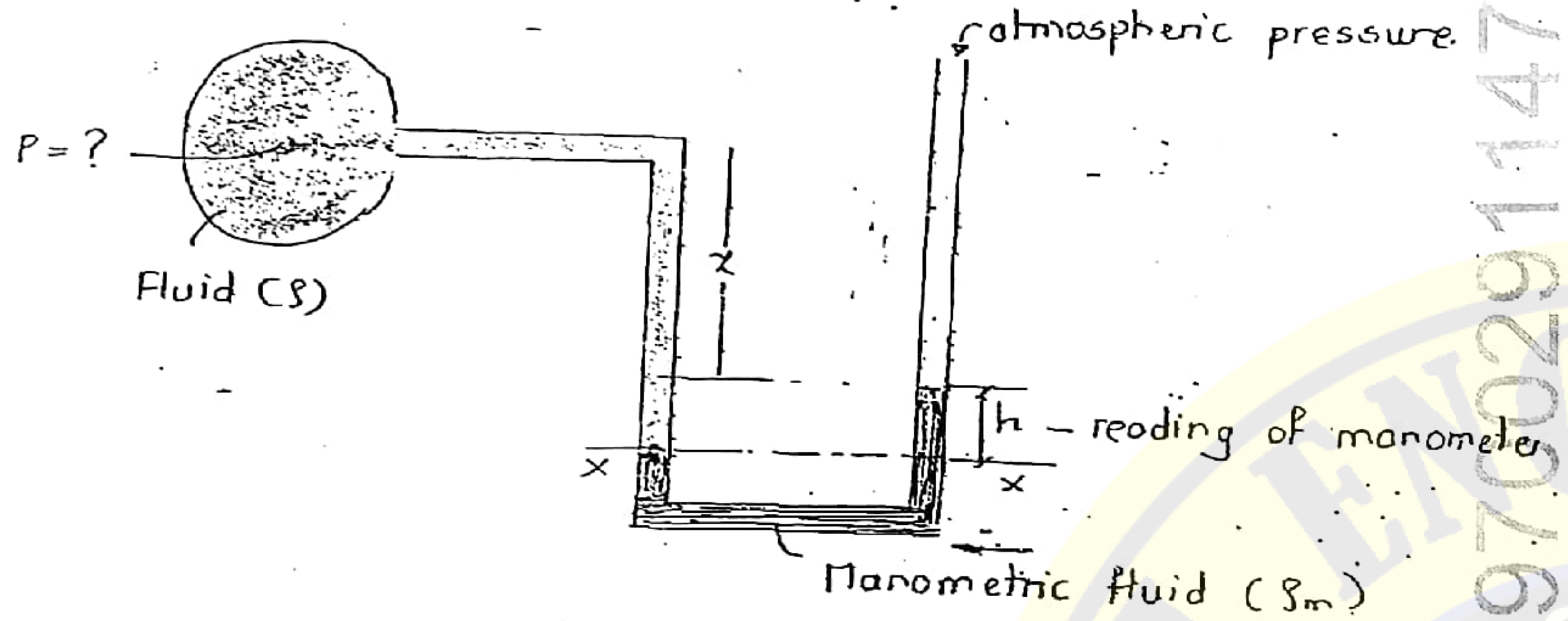
It is a device which is used to measure low, medium and high pressures +ve or -ve gauge pressures of liquids and gases both.

In manometer the additional fluid is used called as Manometeric fluid (density - ρ_m).

7th Unit of pressure

1 tor = 1 mm Hg in Barometer.

a) Simple U-tube manometer:

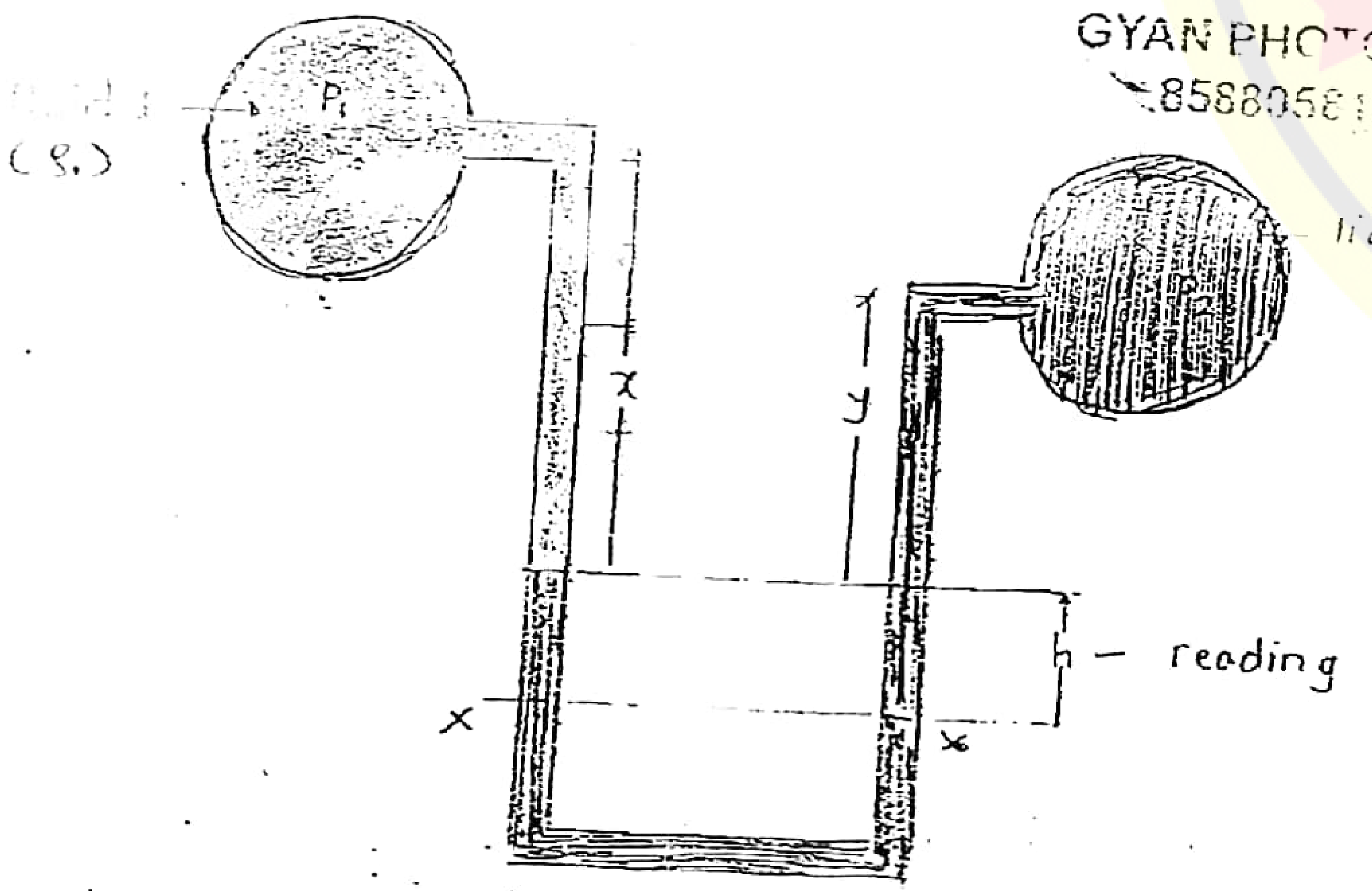


$$P + (x+h)\rho g - \rho_m \cdot g \cdot h = 0$$

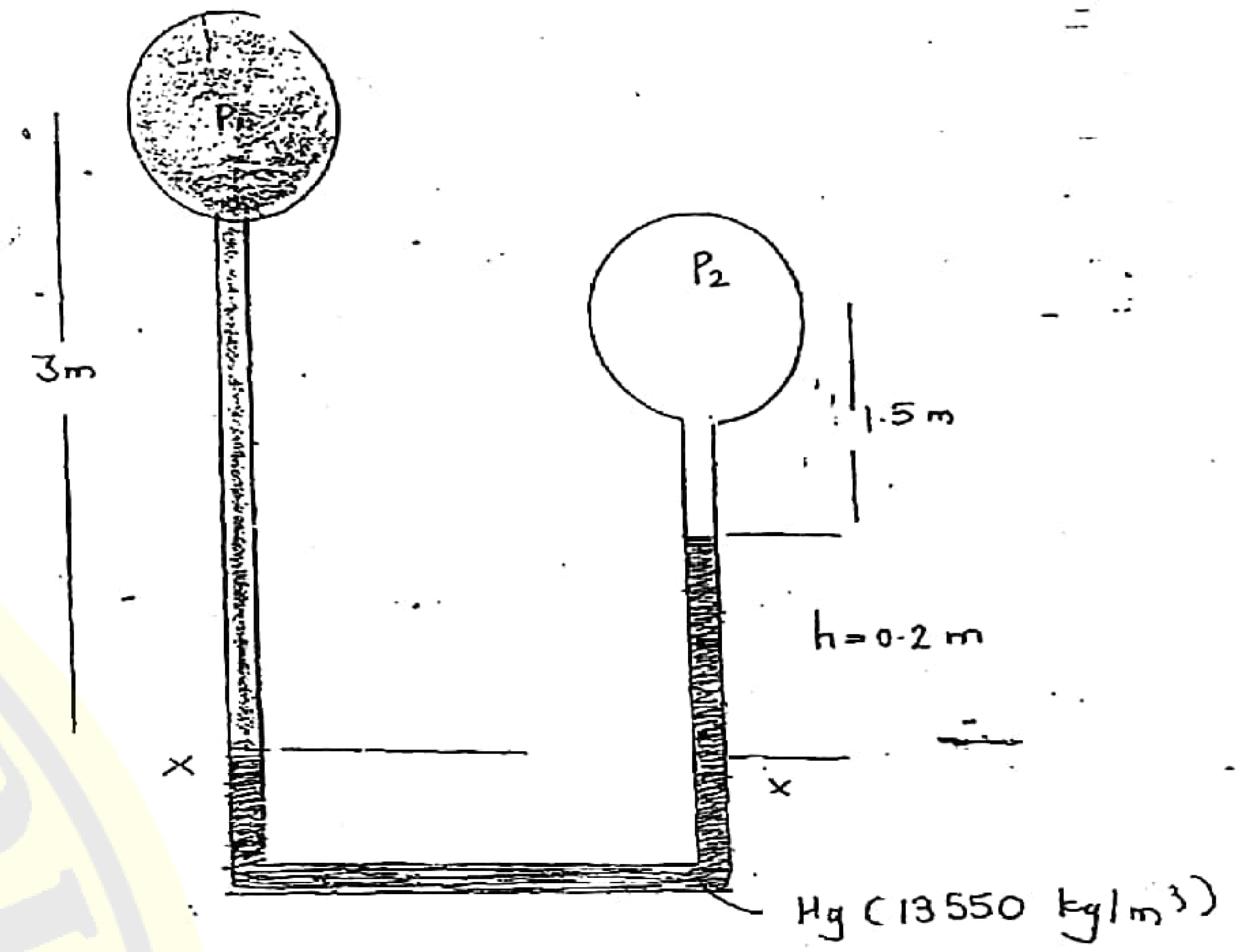
$$P = \rho_m h - (x+h)\rho \quad \text{(gauge pressure)}$$

b) Differential U-tube manometer:

It measures the pressure difference between two points

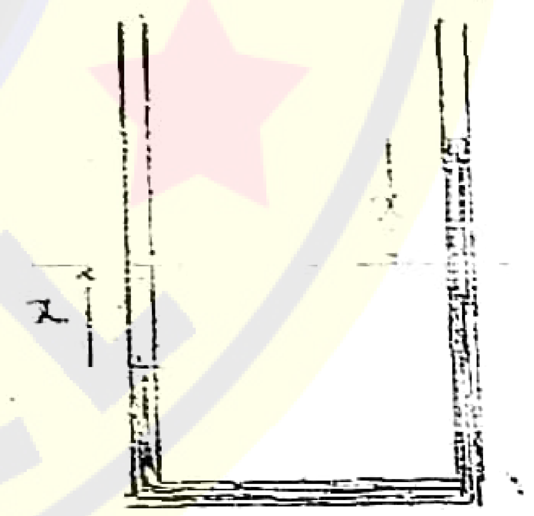


$$P_1 + x\rho_1 g + h\rho_m g - (h+y)\rho_2 g = P_2$$



$$P_1 + (3\times 900\times g) - (0.2\times 13550\times g) - (1.5\times 1000\times g) = P_2$$

$$P_1 - P_2 =$$



$\cancel{P_1 + (2\cdot}$ $\because x + x = 0.2$

$x = 0.1$

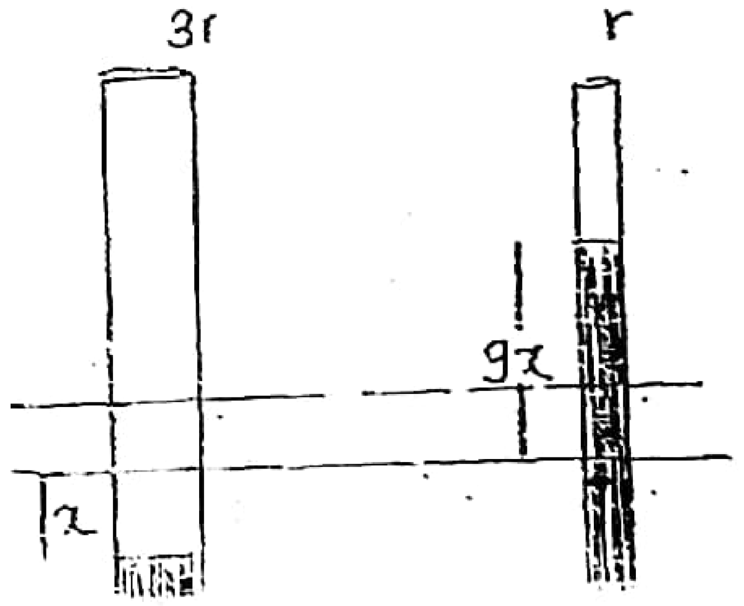
$\therefore$ oil column $= 3 - 0.1 = 2.9$

water $= 1.6$ m

$$P_1 + (2.9\times 900\times g) - (1.6\times 1000\times g) = P_2 + \Delta P$$

$$\Delta P =$$

Variations:

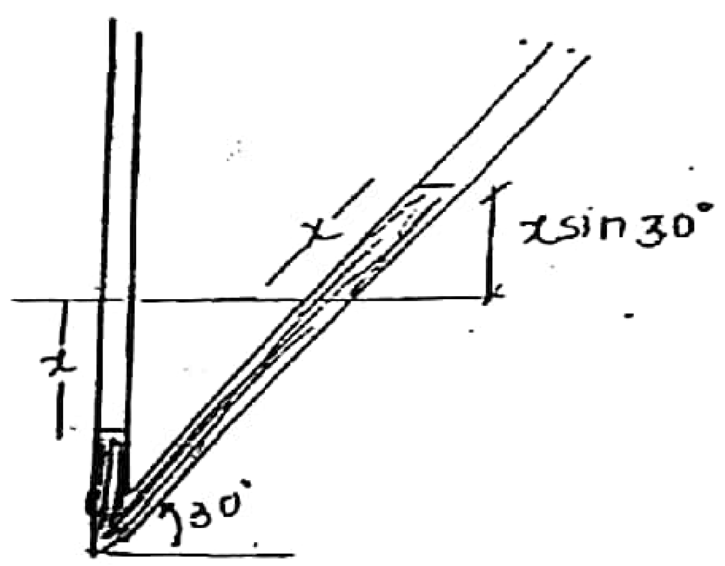


$9x + x = 0.2$

$x = 0.02$

Oil column $= (3 - x) = 2.98$ m

water $= (1.5 + 9x) = 1.68$ m

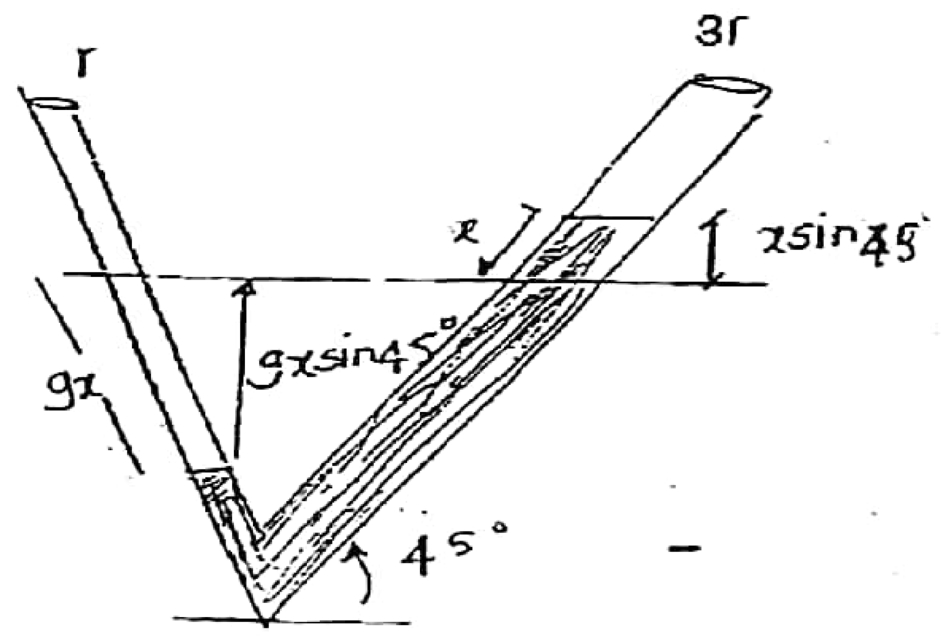


$x \sin 30 + x = 0.2$

$x =$

oil : $(3 - x)$

water : $(1.5 + x \sin 30) =$



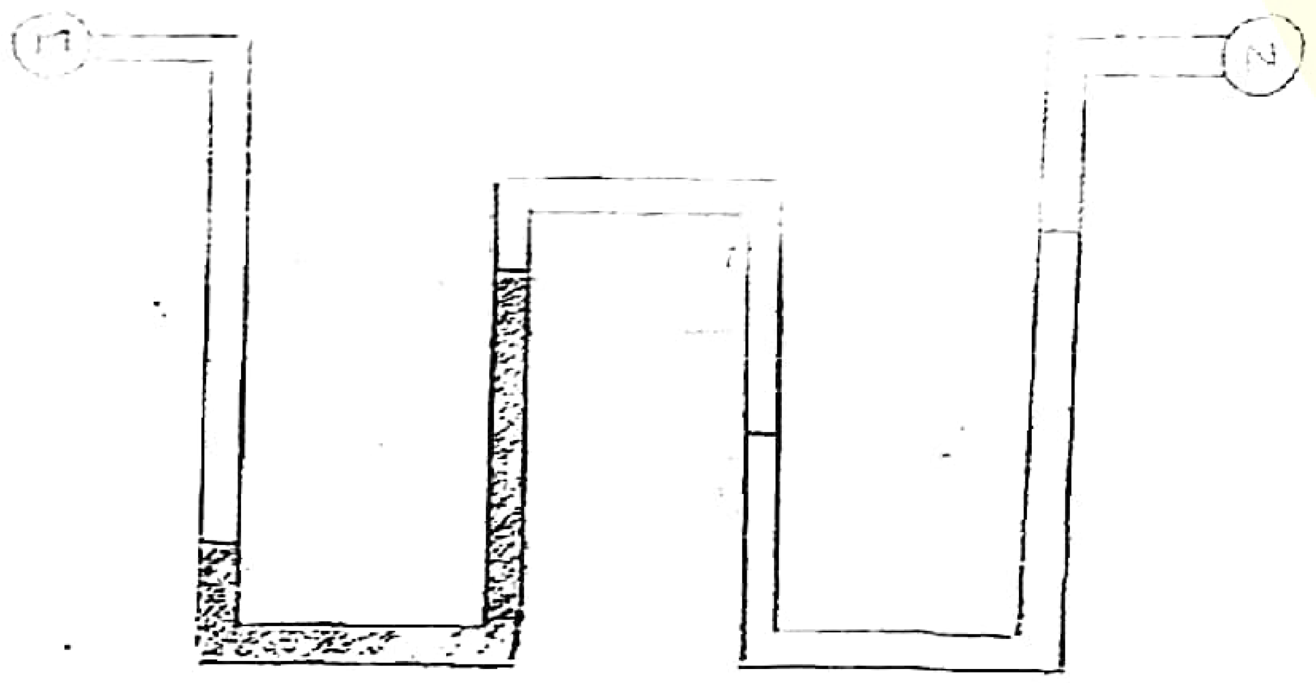
$x \sin 45° + \cancel{x} \; gx \sin 45° = 0.2$

$x =$

oil : $(3 - gx \sin 45) =$

water : $(1.5 + x \sin 45) =$

Q. Find difference of pressure $(P_M - P_N)$



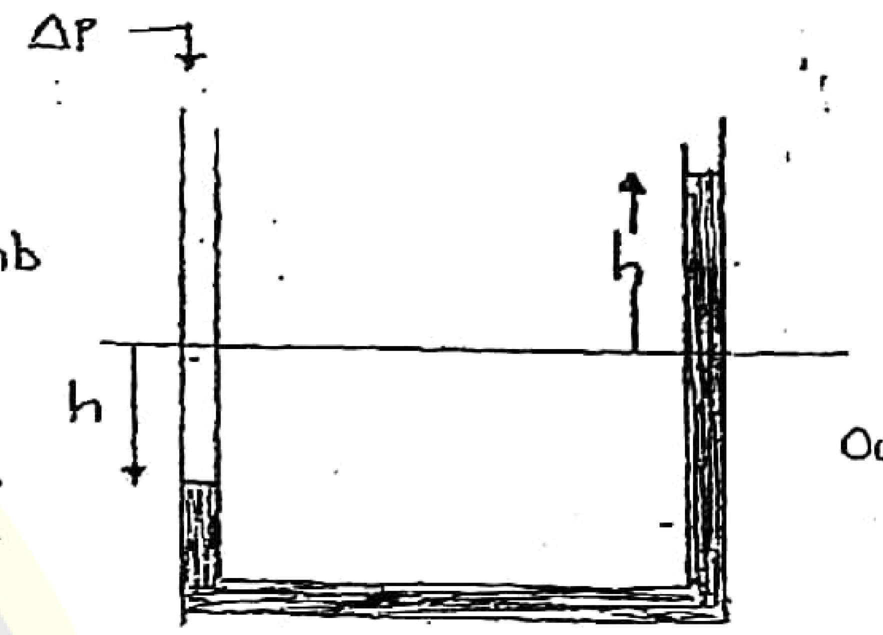
$P_M + (36 \times 800 \times g) - (12 \times 13600 \times g) + (12 \times 800 \times g)$

$- (15 \times 13600 \times g) - (15 \times 800 \times g) = P_N$

$P_M - P_N =$

Sensitivity of Manometer.

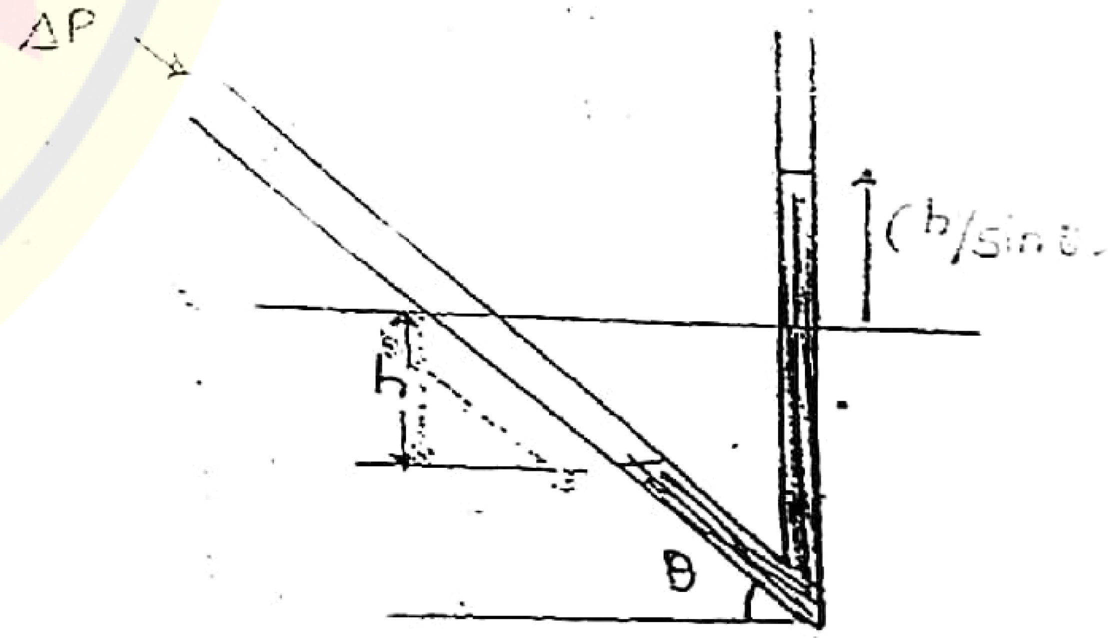
The extent of reading in the output limb of the manometer for a given pressure difference in input limb is known as Sensitivity of Manometer.



'h' is called Sensitivity of Manometer for applied pressure ΔP (excess pressure)

(i) To increase sensitivity:

Input limb is inclined.

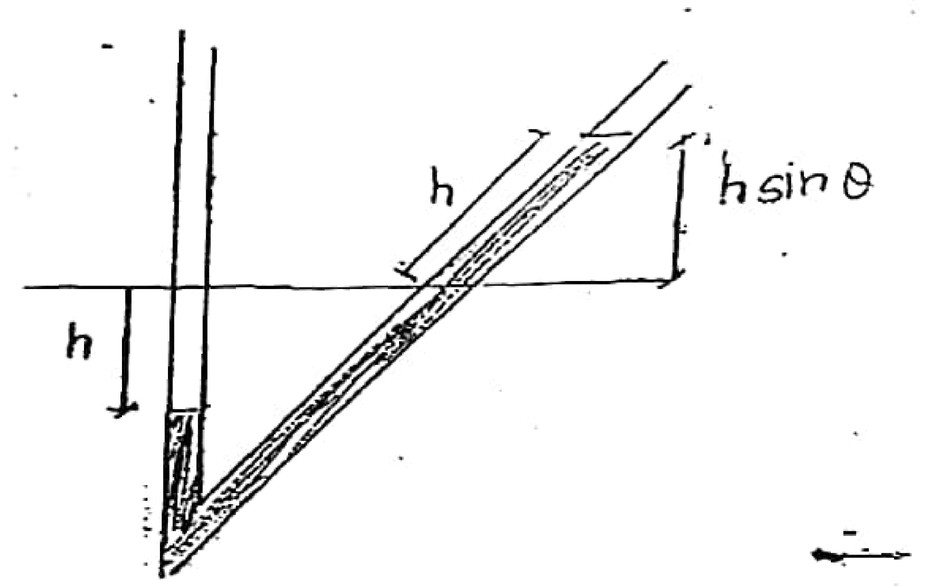


Sensitivity is increased by $\left(\frac{1}{\sin\theta}\right)$ times.

When the readings are very small, the sensitivity of the manometer is increased by inclining the i/p limb.

(ii) To decrease sensitivity:

The output limb is inclined.



Sensitivity is decreased by (sin θ) times.

When the pressure difference (readings) is more than the sensitivity of manometer is decreased. (c/s areas can also be increased to reduce sensitivity.)

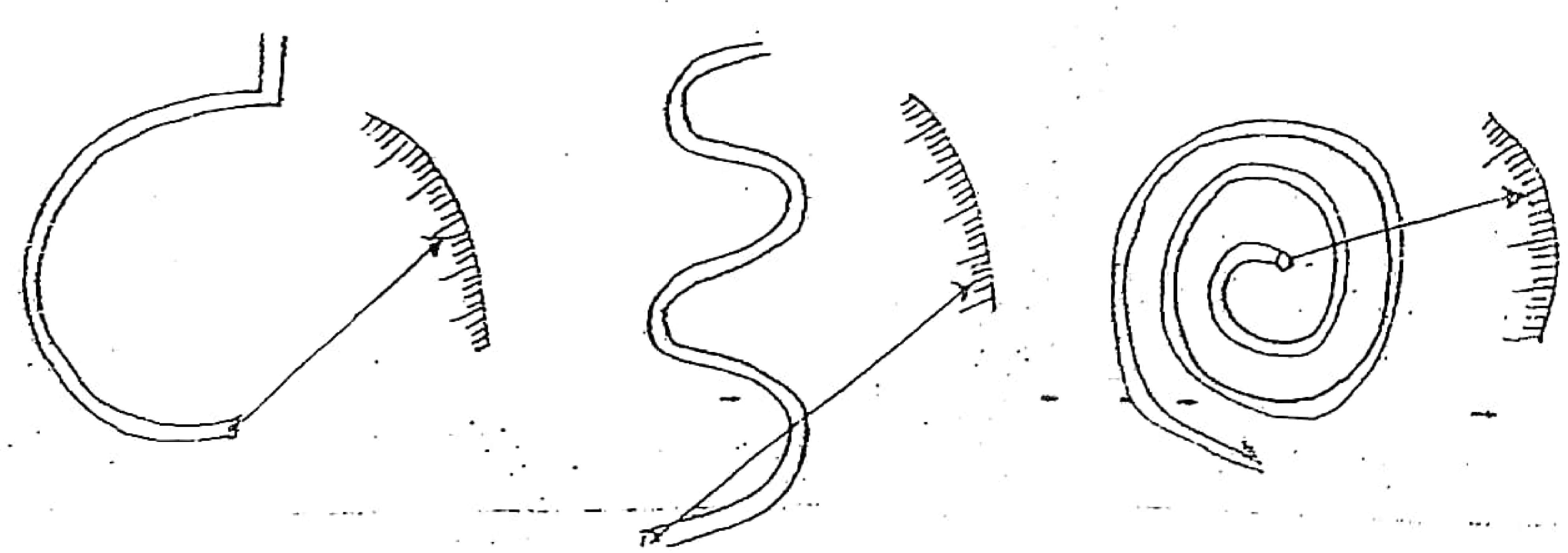
Modern pressure measurement devices:

1. Bourdon gauge:

There is small flexible tube inside this gauge.

It measures the pressure above atmospheric pressures only (gauge pressures)

The property used in pressure measurement is Flexibility.



2. Strain gauge transducer.

It contains very small chips of small materials (alloys). The small wire coming out of two chips is attached to Data Aquastion System (DAS) to measure the strain developed.

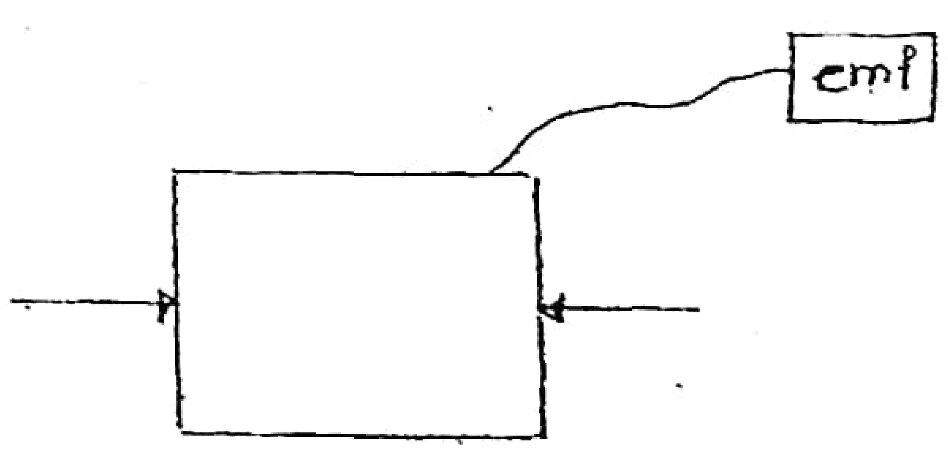
The shock developed are measured in terms of strain. (strain → stress → pressure)

e.g. On road testing of vehicles

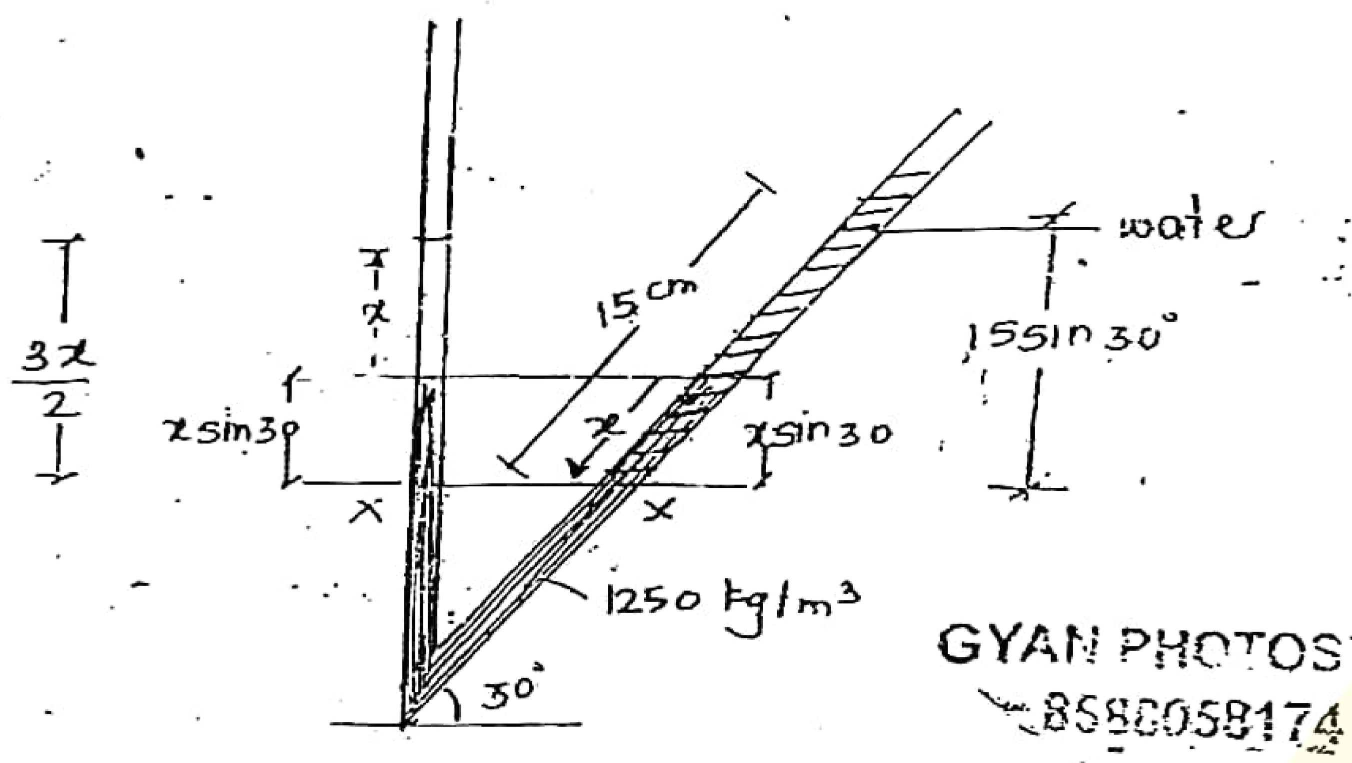
3. Piezo-electric transducers:

Piezo- pressure, electric - emf

The emf is developed in the small transducers as the pressure is applied. This property is found only in Quartz and Rochelle salt.



Q. 19. (page 13)



$$\cancel{P_{atm}} + \left(\frac{3x}{2}\right) \times 1250 \times \cancel{g} - 0.075 \times 1000 \times \cancel{g} = \cancel{P_{atm}}$$

$x = 4$ cm

1 atm = 101325 Pa

=

1 $\frac{kg.f}{cm^2}$ = 0.967 atm

1 Pa = [illegible] $\times 10^{-6}$ atm

1 bar = [illegible] atm

1 tor = 1.31×10^{-3} atm = 1 mm. of Hg.

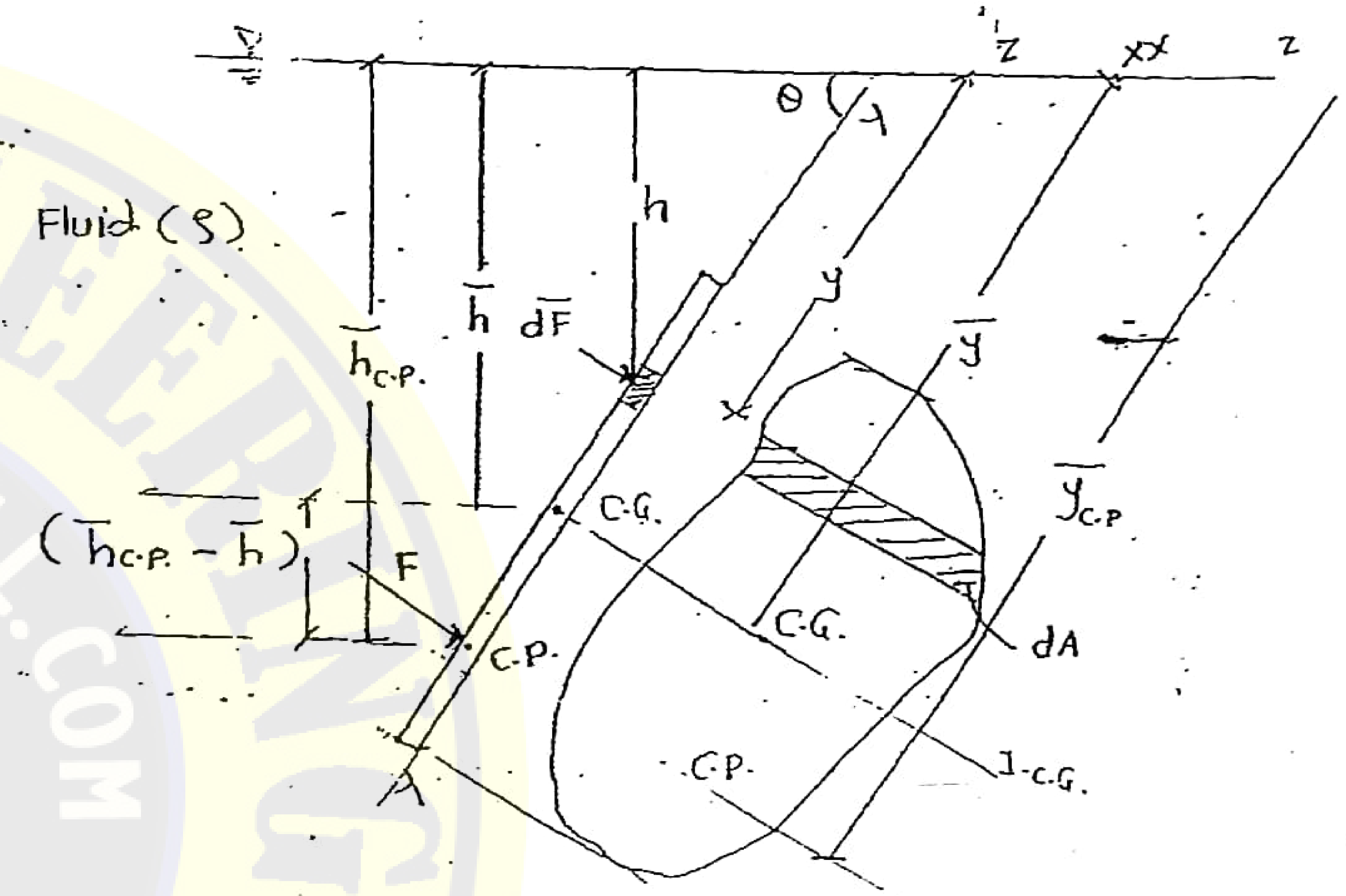
1 PSI = 0.068 atm

1 m water = 0.097 atm

Pa < tor < PSI < water < $\frac{kg.f}{cm^2}$ < bar < atm

FLUID STATICS

Hydrostatic pressure on the plane surface.



$$\sin\theta = \frac{\bar{h}}{\bar{y}} = \frac{h}{y} = \frac{h_{c.p}}{y_{c.p}}$$

Hydrostatic force:

$$F = \int dF$$

$$= \int P \cdot dA$$

$$= \int \rho g h \cdot dA$$

$$= S g \int y \cdot \sin\theta \cdot dA$$

$$= S g \sin\theta \int y \cdot dA$$

$$= S g \sin\theta \cdot \bar{y} \cdot A \qquad \because \bar{y} = \frac{\int y \cdot dA}{A}$$

The resultant force is passing through the point on the body known as centre of pressure.

Location of C.P.

$$\int dF \cdot y = F \cdot \bar{y}_{cp} \quad \text{(varignon's theorem)}$$

$$\rho g \sin\theta \cdot \int y^2 \, dA = \rho g \bar{h} \cdot A \cdot \frac{\bar{h}_{cp}}{\sin\theta}$$

$$\bar{h}_{cp} = \frac{\sin^2\theta}{\bar{h} \cdot A} \cdot I_{xx} \qquad \int y^2 \, dA = I_{xx}$$

$$= \frac{\sin^2\theta}{\bar{h} \cdot A} \cdot (I_{c.g} + \bar{y}^2 A)$$

$$I_{xx} = I_{c.g} + \bar{y}^2 A$$

$$= \frac{\sin^2\theta}{\bar{h} \cdot A} \cdot \left(\frac{\bar{h}^2}{\sin^2\theta} \cdot A + I_{c.g.}\right)$$

$$\boxed{\bar{h}_{cp} = \bar{h} + \frac{I_{c.g.} \sin^2\theta}{\bar{h} \cdot A}}$$

Note

$$(\bar{h}_{cp} - \bar{h}) = \frac{I_{c.g.} \sin^2\theta}{\bar{h} \cdot A}$$

$I_{c.g.}$, A and θ for given surface are constant.

If surface is taken to more depth, $\bar{h}$ increases so $(\bar{h}_{cp} - \bar{h})$ will decrease. i.e. Centre of pressure (C.P.) shift towards the centre of gravity (C.G.)

If surface is taken to ∞ depth, $\bar{h} \to \infty$.

$$\bar{h}_{cp} - \bar{h} = 0$$

$$\bar{h}_{cp} = \bar{h} \qquad \text{or}$$

$$\bar{h}_{cp} = \bar{h} + \frac{I_{c.g.} \sin^2\theta}{\bar{h} \cdot A}$$

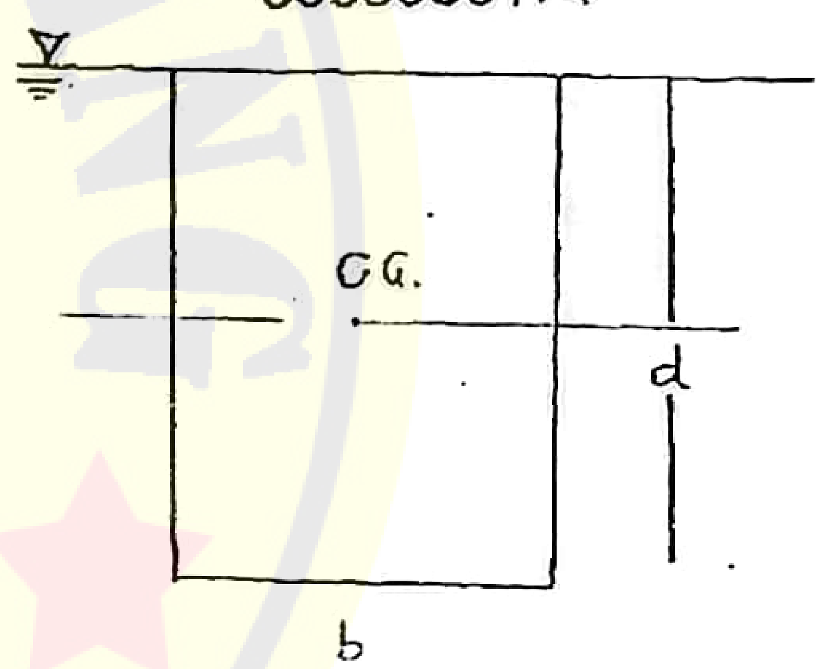
For horizontal surface, $\theta = 0$

$$\bar{h}_{cp} = \bar{h}$$

For vertical surface, $\theta = 90^\circ$

$$\bar{h}_{cp} = \bar{h} + \frac{I_{c.g.}}{\bar{h} \cdot A}$$

Few vertical surfaces:



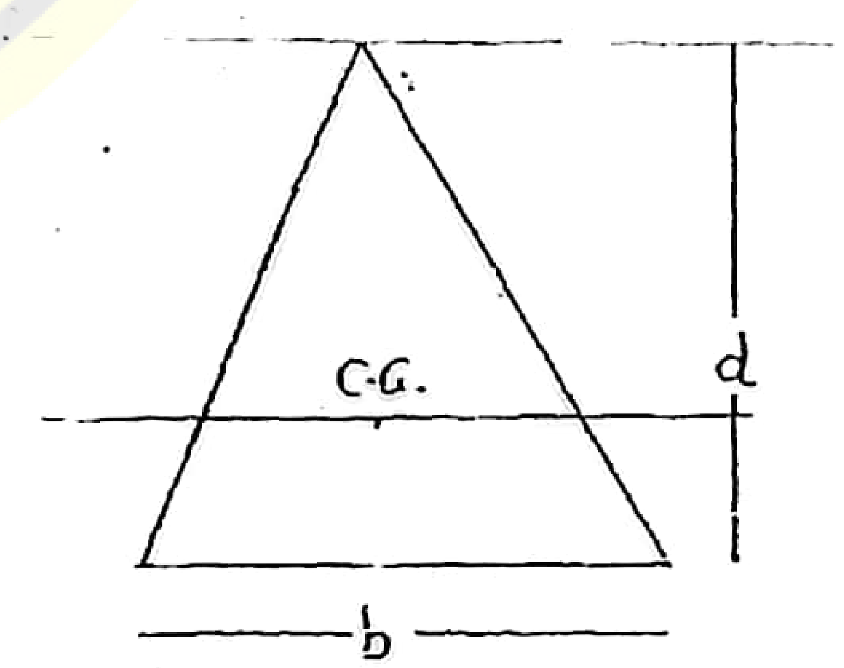
$A = b.d.$

$\bar{h} = d/2$

$I_{c.g.} = \frac{bd^3}{12}$

$$\bar{h}_{c.p.} = \bar{h} + \frac{bd^3/12}{d/2 \cdot bd}$$

$$\bar{h}_{c.p.} = \frac{2}{3} d$$



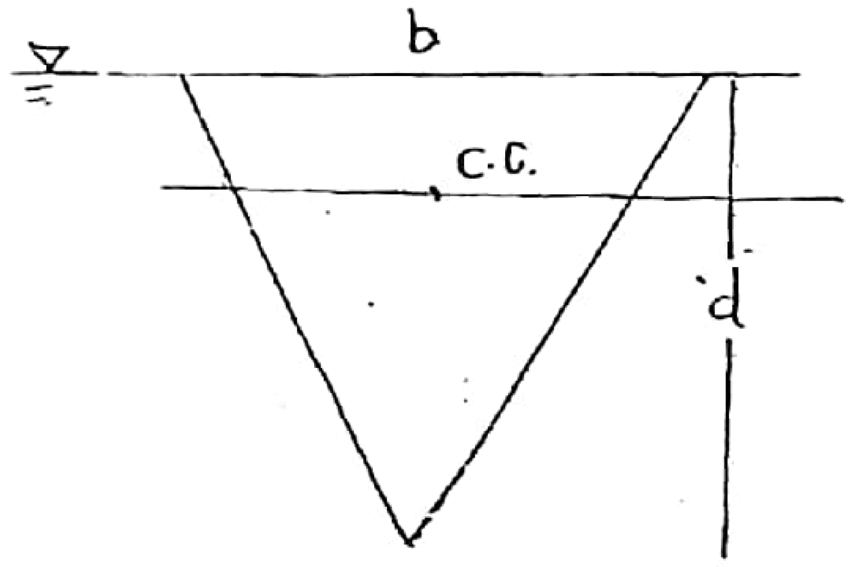
$A = \frac{bd}{2}$

$\bar{h} = \frac{2}{3} \cdot d$

$I_{c.g} = \frac{bd^3}{36}$

$$\bar{h}_{c.p.} = \frac{2}{3} d + \frac{bd^3/36}{2/3 \cdot d \times bd/2}$$

$$\bar{h}_{cp} = \frac{3}{4} d$$



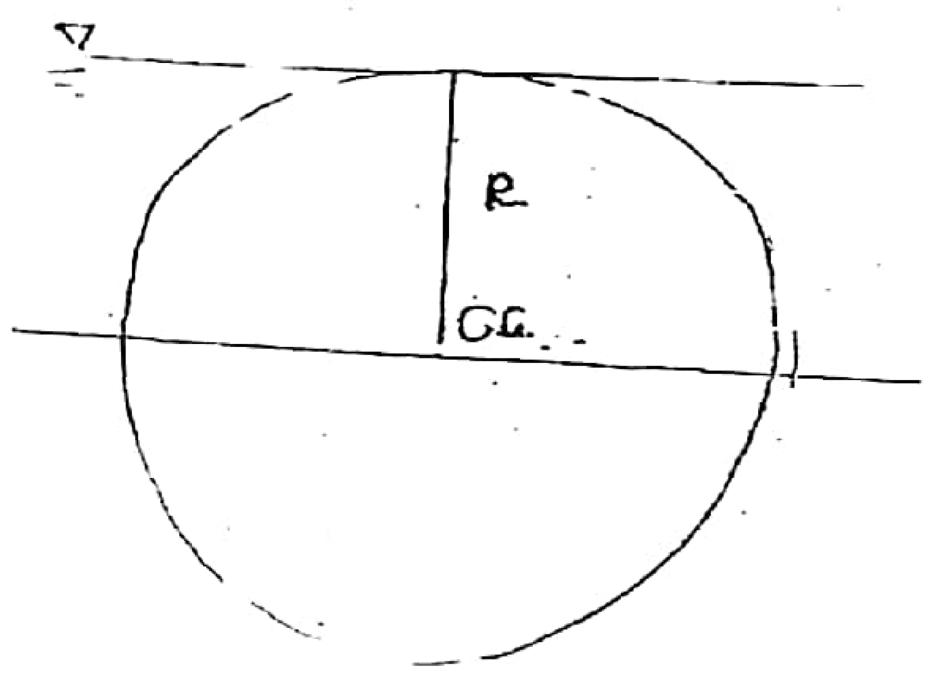
$A = \frac{bd}{2}$

$\bar{h} = \frac{2d}{3}$

$I_{c.G.} = \frac{bd^3}{36}$

$\bar{h}_{c.p.} = \frac{d}{3} + \frac{bd^3/36}{d/3 \times bd/2}$

$= \frac{d}{2}$



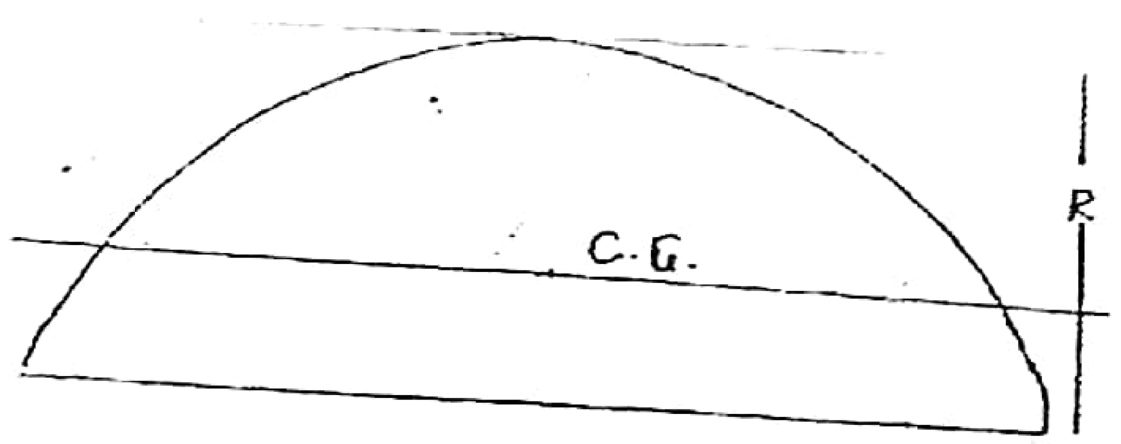
$A = \pi R^2$

$\bar{h} = R$

$I_{c.G} = \frac{\pi R^4}{4}$

$\bar{h}_{c.p.} = R + \frac{\pi R^4/4}{R \cdot \pi R^2}$

$= \frac{5}{4} R$



$A = \frac{\pi R^2}{2}$

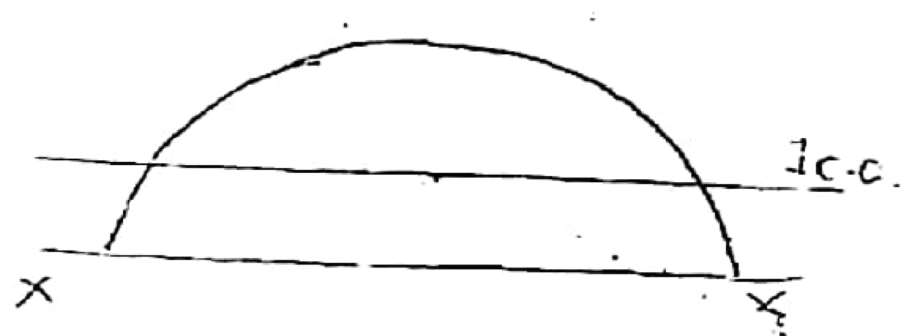
$\bar{h} = \left(R - \frac{4R}{3\pi}\right)$

$I_{C.G.} = 0.1097\, R^4$

$I_{c.G} = R^4 \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$

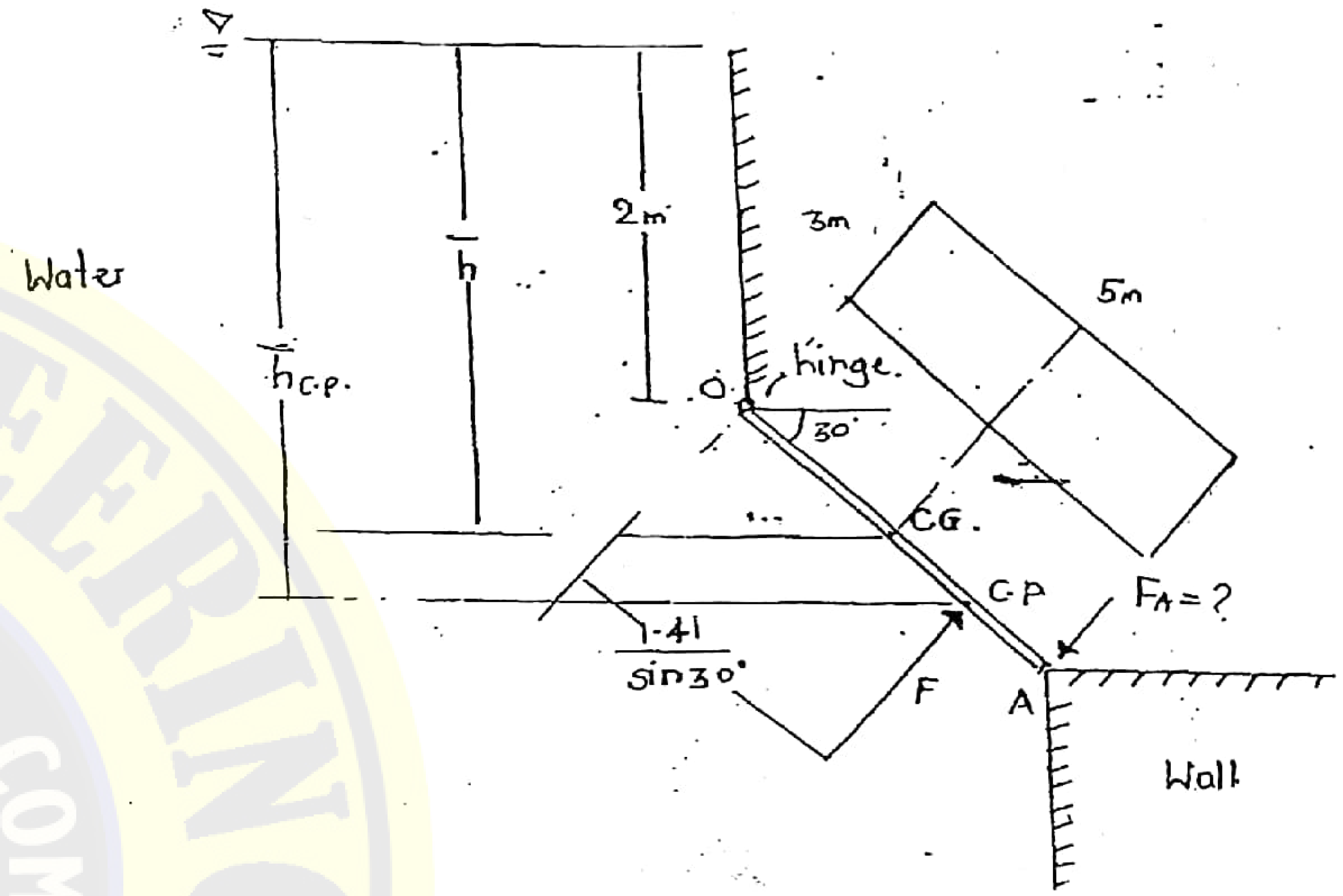
$\bar{h}_{c.p.} = \left(R - \frac{4R}{3\pi}\right) + \frac{0.1097\, R^4}{\frac{\pi R^2}{2}\left(R - \frac{4R}{3\pi}\right)}$

$= 0.5755 R + 0.1219 R$



$I_{xx} = I_{c.G} + \left(\frac{4R}{3\pi}\right)^2 \cdot \frac{\pi R^2}{2}$

Q. Find the force F_A in order to keep th gate to be closed



Hydrostatic force:

$\bar{h} = 2 + 2.5 \sin 30°$

$\bar{h} = 3.25$ m

$\theta = 30°$

$A = 5 \times 3 = 15\ m^2$

$I_{c.G} = \frac{3(5)^3}{12} = 31.25\ m^4$

$F = \rho g \bar{h} \cdot A$

$= (1000 \times 9.8 \times 3.25 \times 15)$

$= 478.237$ kN.

$\bar{h}_{c.p.} = \bar{h} + \frac{I_{CG} \sin^2\theta}{\bar{h} \cdot A}$

$= 3.25 + \frac{31.25 \times \sin^2(30)}{}$ -

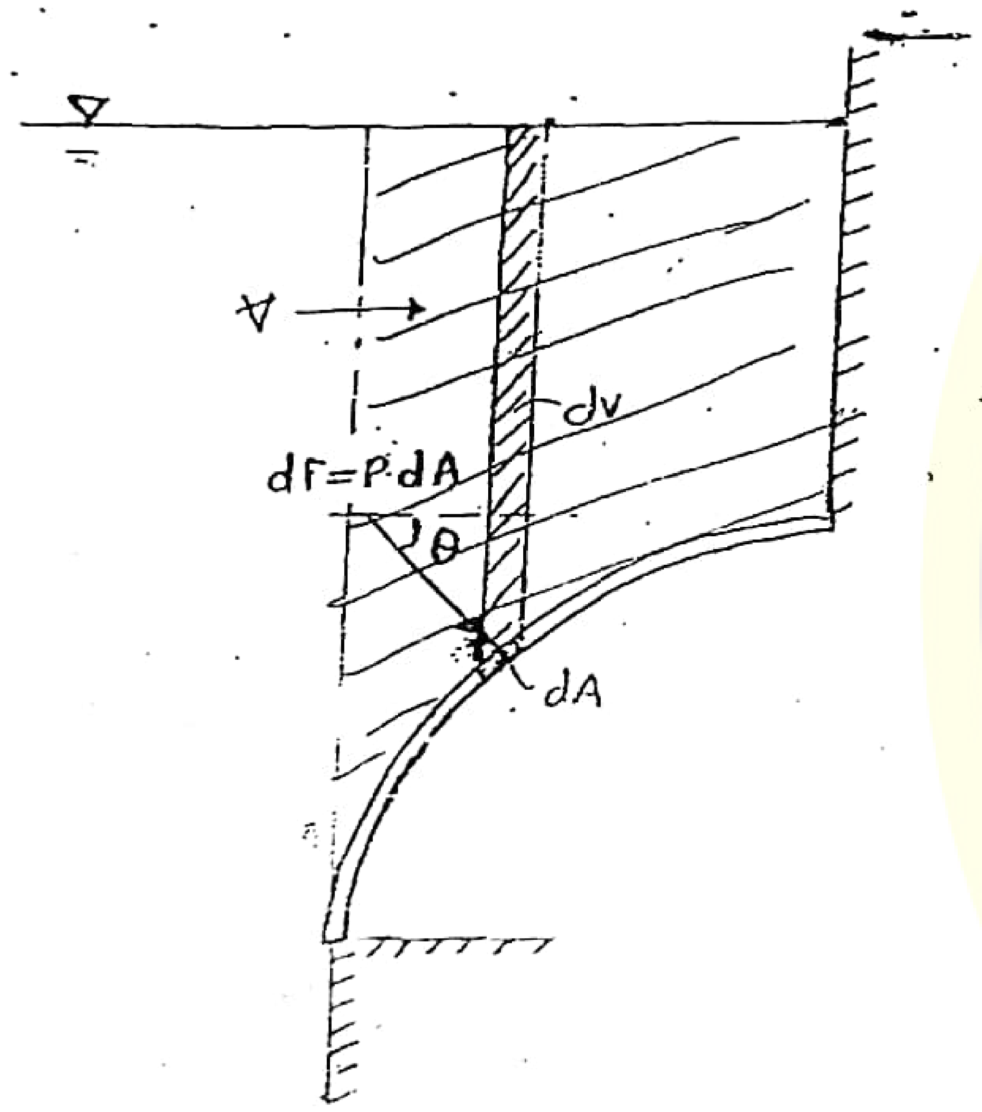
Take $M_O = 0$

$F \times 2.82 = F_A \times 5$

$F_A = \dfrac{478.237 \times 2.82}{5}$ $\quad \because \dfrac{1.41}{\sin 30} = 2.82\,m$

$F_A = 269.73$ kN

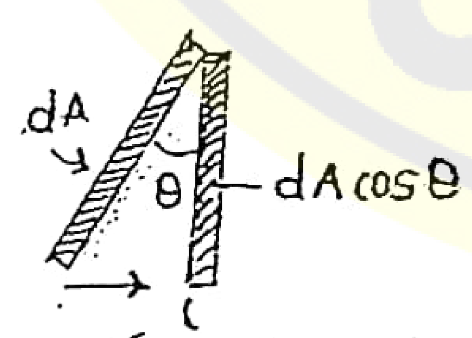
Hydrostatic forces on curved surface:



Horizontal analysis.

$F_H = \int P \cdot dA \cdot \cos\theta$

$= \int \rho g h \cdot dA_H$



(Area projected in horizontal direction (Area in vertical plane)

$\because (dA \cos\theta = dA_H)$

$F_H = \rho g \bar{h} \cdot A_H$

where A_H - projected area of curved surface in horizontal direction (on vertical plane)

Vertical analysis:

$F_V = \int P \cdot dA \sin\theta$

$= \int \rho g h \cdot dA \cdot \sin\theta$

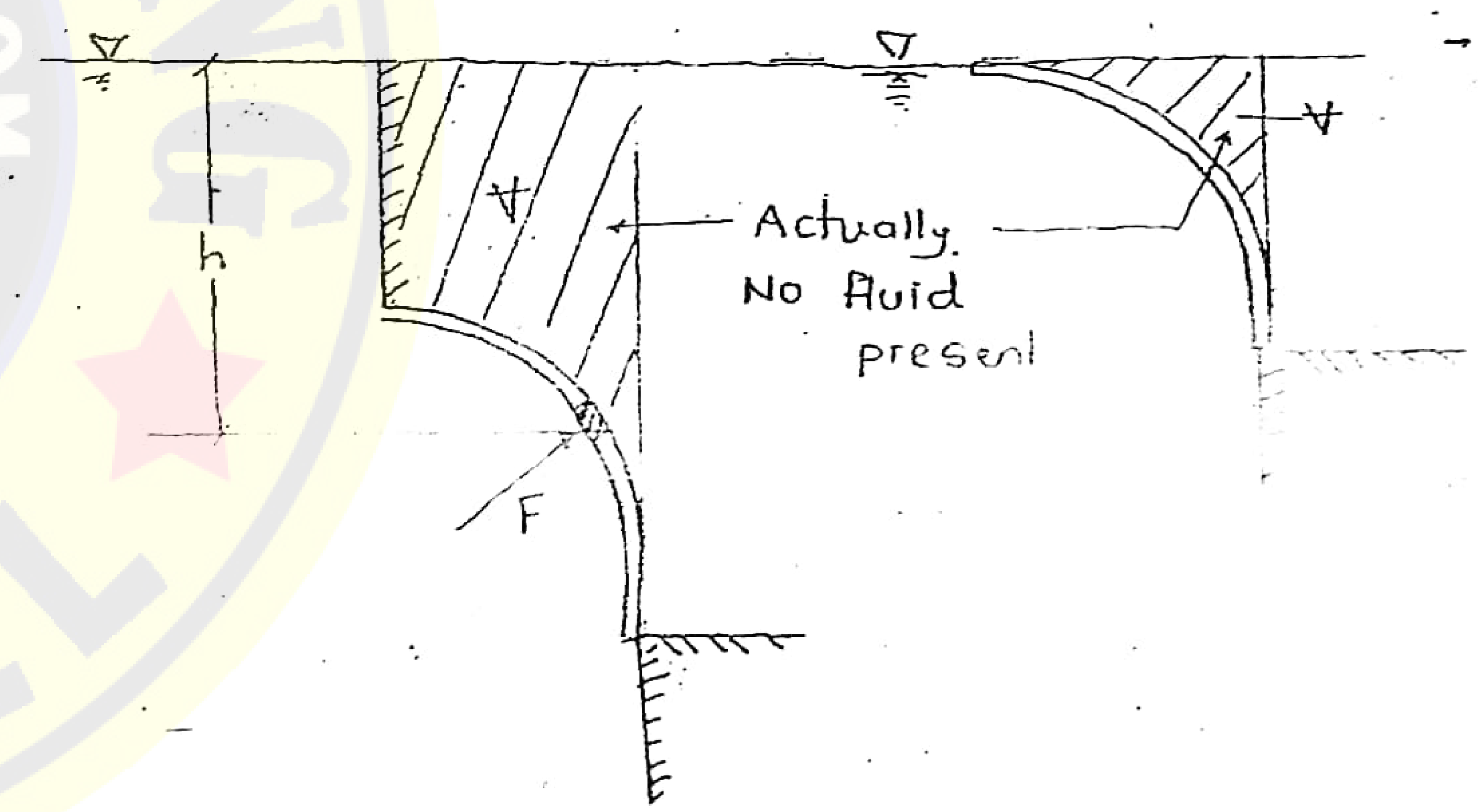
$= \rho g \int h \cdot dA \cdot \sin\theta$

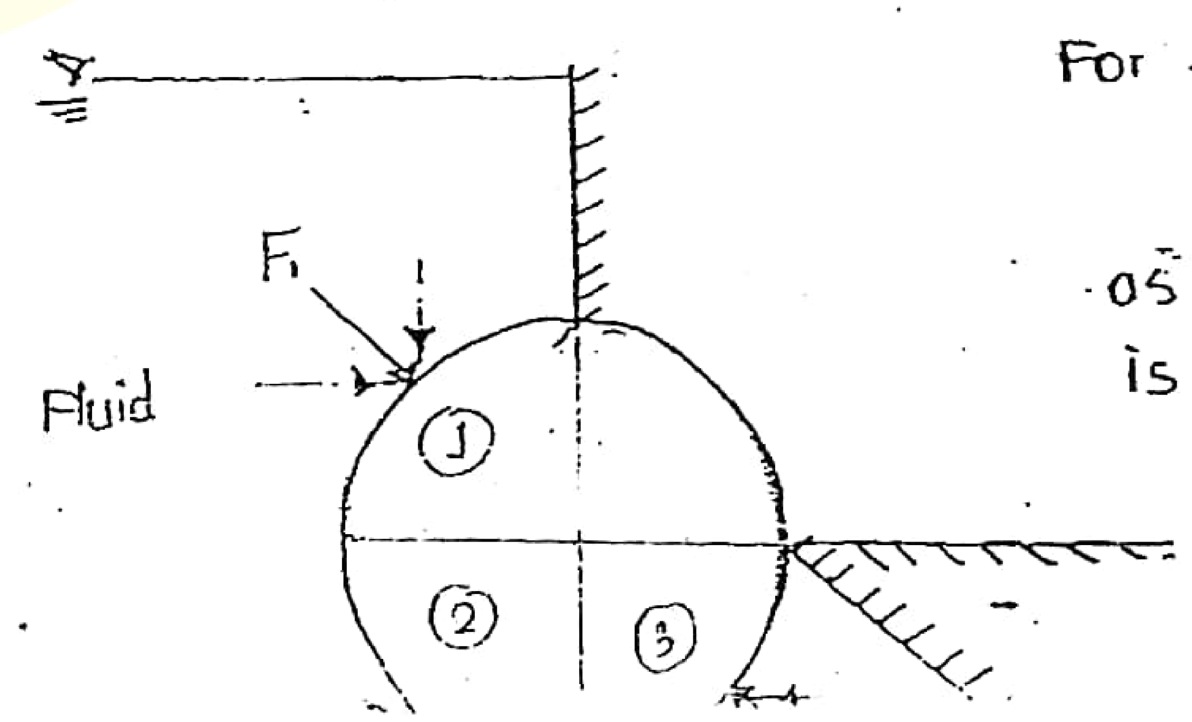
$= \rho g \int d\forall$ $\quad * \ h \cdot dA \cdot \sin\theta = d\forall$

$F_V = \rho g \cdot \forall$

where

$\forall$ - volume above the curved surface upto free surface.





For horizontal analysis

① + ② can be taken together as direction of horizontal force is same. (→)

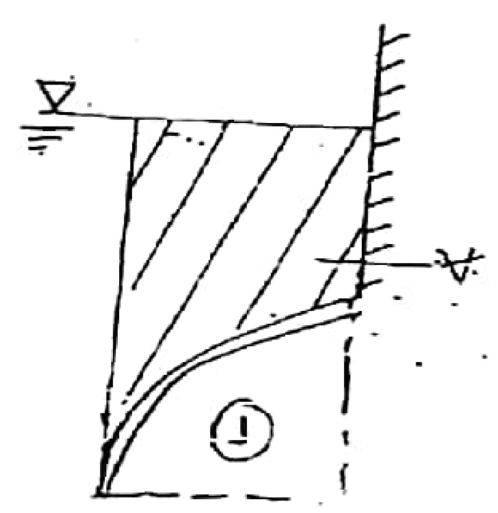
③ can be taken seperately) -(←)

For vertical analysis

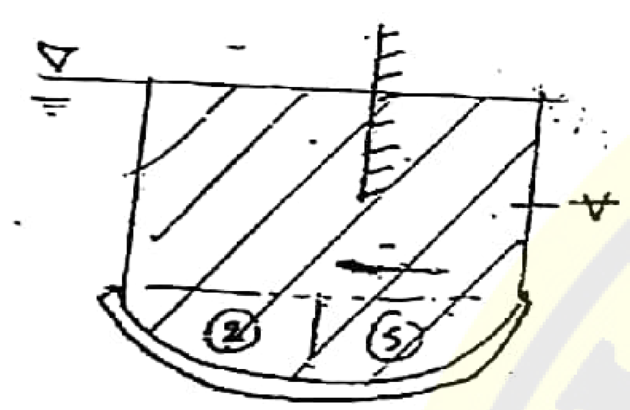
① can be taken alone (↓)

② and ③ can be taken together (↑)

For ① ↓

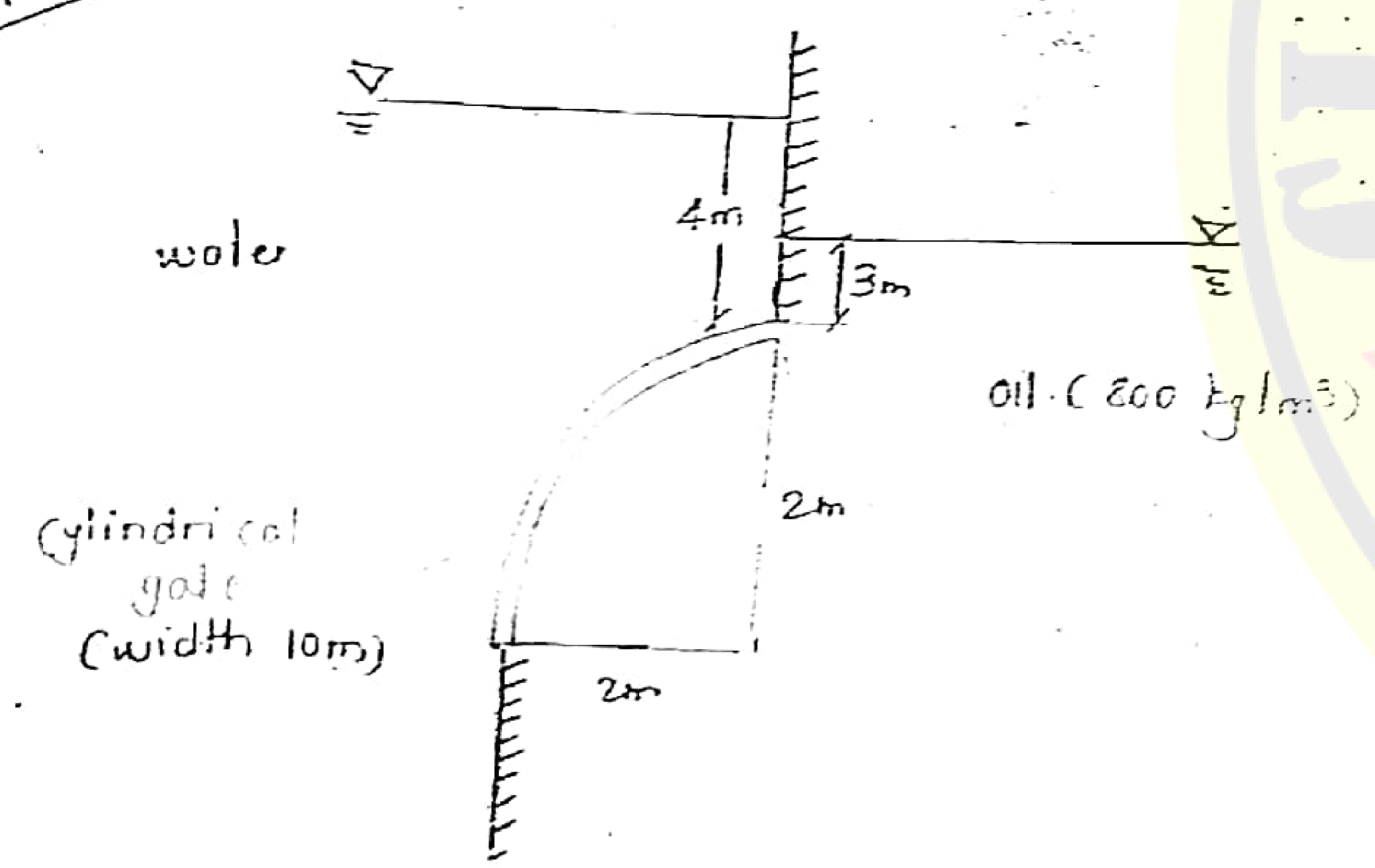


For ② & ③ ↑

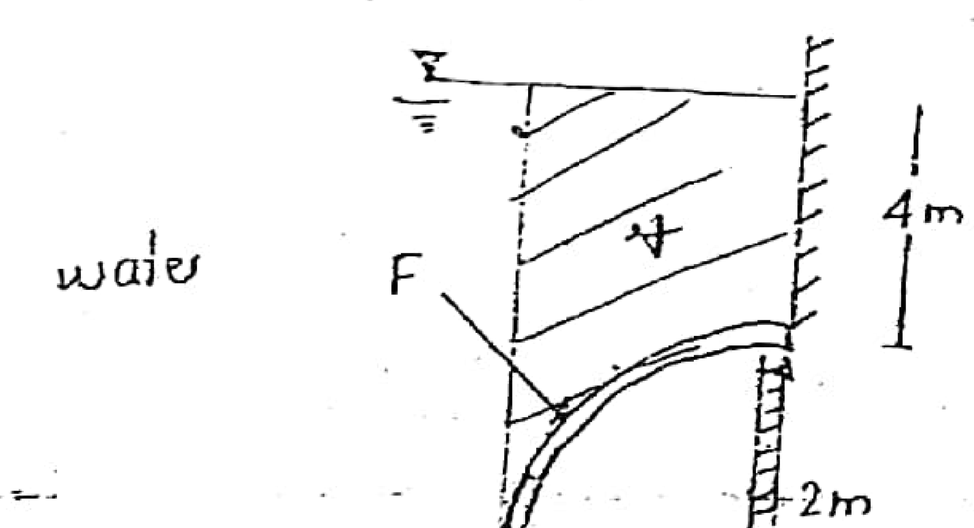


Q. Find the total hydrostatic force on the curved surface.

20 Marks



Force from the water:



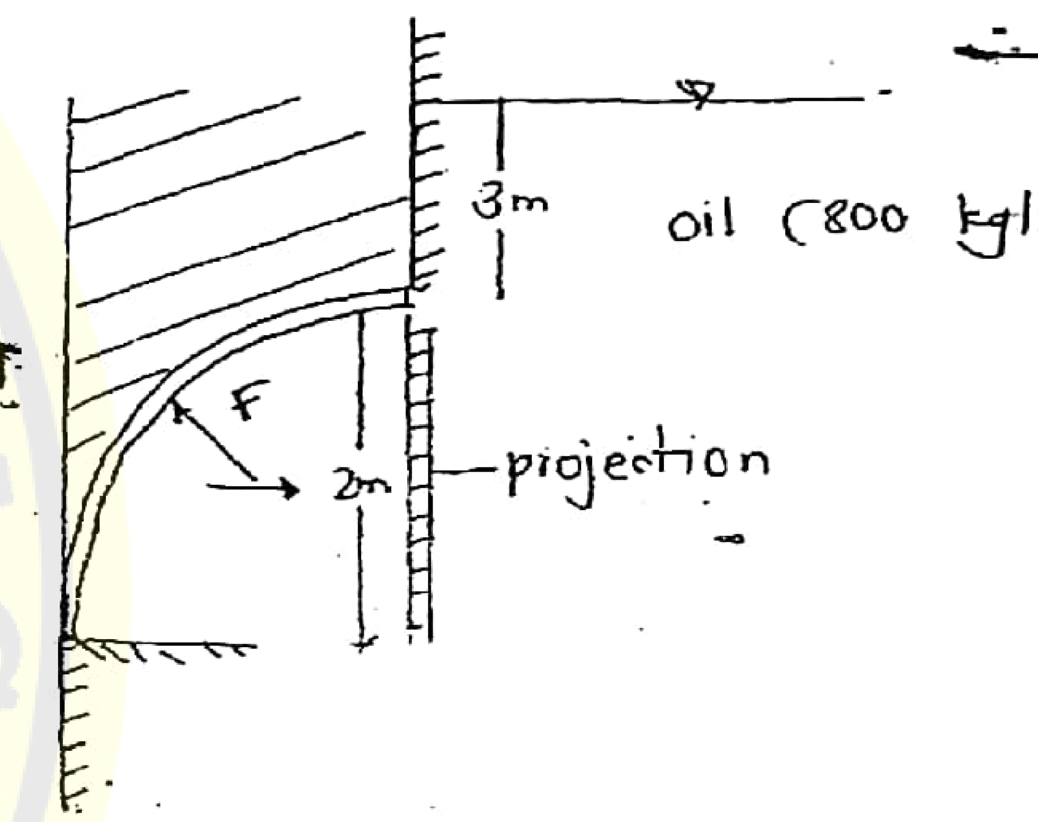
$F_{H_1} = (1000 \times 9.81 \times 5 \underbrace{(2 \times 10)}_{A_H})\ N \rightarrow$

$= 981\ kN$

$F_{V_1} = 1000 \times 9.81 \times \underbrace{\left[(6 \times 2) - \frac{\pi (2)^2}{4} \right]}_{\forall} \times 10 \downarrow$

$= 869\ kN$

Force from oil.



$F_{H_2} = (800 \times 9.81 \times 4 \underbrace{(2 \times 10)}_{A_H}) \leftarrow$

$= 672.84\ kN$

$F_{V_2} = 800 \times 9.81 \times \underbrace{\left[(5 \times 2) - \frac{\pi (2)^2}{4} \right]}_{\forall} \times 10 \uparrow$

$= 537.69\ kN$

$F_H = |F_{H_1} - F_{H_2}|$

$= 308.16\ kN$

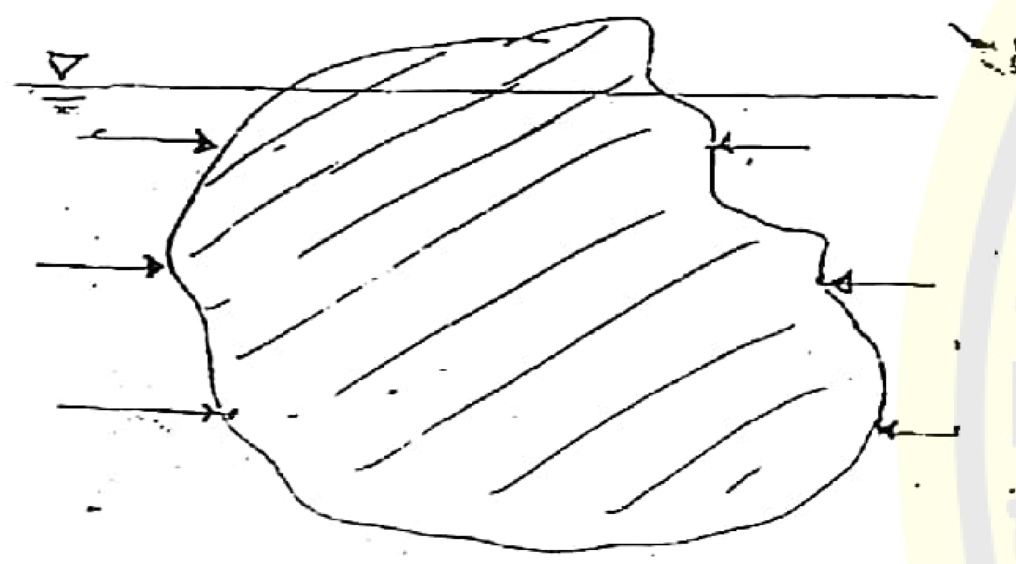
$F_V = |F_{V_1} - F_{V_2}|$

$= 331.31$

$F_R = \sqrt{F_H^2 + F_V^2}$

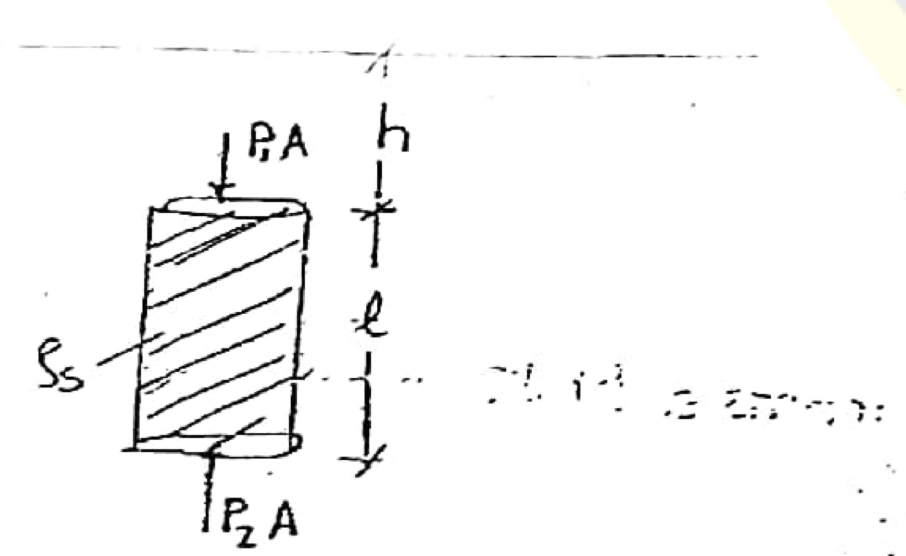
Hydrostatic forces on the bodies :
(Archimedes Principle) - Buoyancy.

"When a body is submerged or emersed fully or partially inside a static fluid, then the resultant hydrostatic force acts on the body in the vertical upward direction. This force is known as Buoyant force, or upthrust. and the value of this force is exactly same as the weight of the displaced fluid by the body, when it is submerged."



Net horizontal hydrostatic force on body is zero. and net vertical hydrostatic force is vertical upward.

Fluid (ρ)



Hydrostatic force,

$$F_B = P_2 A - P_1 A$$
$$= [\rho g (h + l) - \rho g h] A$$
$$F_B = \rho g A l$$

$$\boxed{F_B = \rho g \forall'_{Body}}$$

m - mass of body
m' - mass of submerged body
$\forall_{Body}$ - volume of body
$\forall'_{Body}$ - volume of submerged body
$\forall$ - volume of fluid displaced.

$m' \leq m$ - may be fully or partially submerged
$\forall'_{Body} \leq \forall_{Body}$

$\forall = \forall'_{Body}$ - (Archimedes principle)

$$F_B = \rho \cdot g \cdot \forall$$
$$= \rho \cdot g \cdot V_{Body}'$$
$$= \rho \cdot g \left(\frac{m'}{\rho_s}\right)$$
$$= \frac{m' g}{\left(\frac{\rho_s}{\rho}\right)}$$

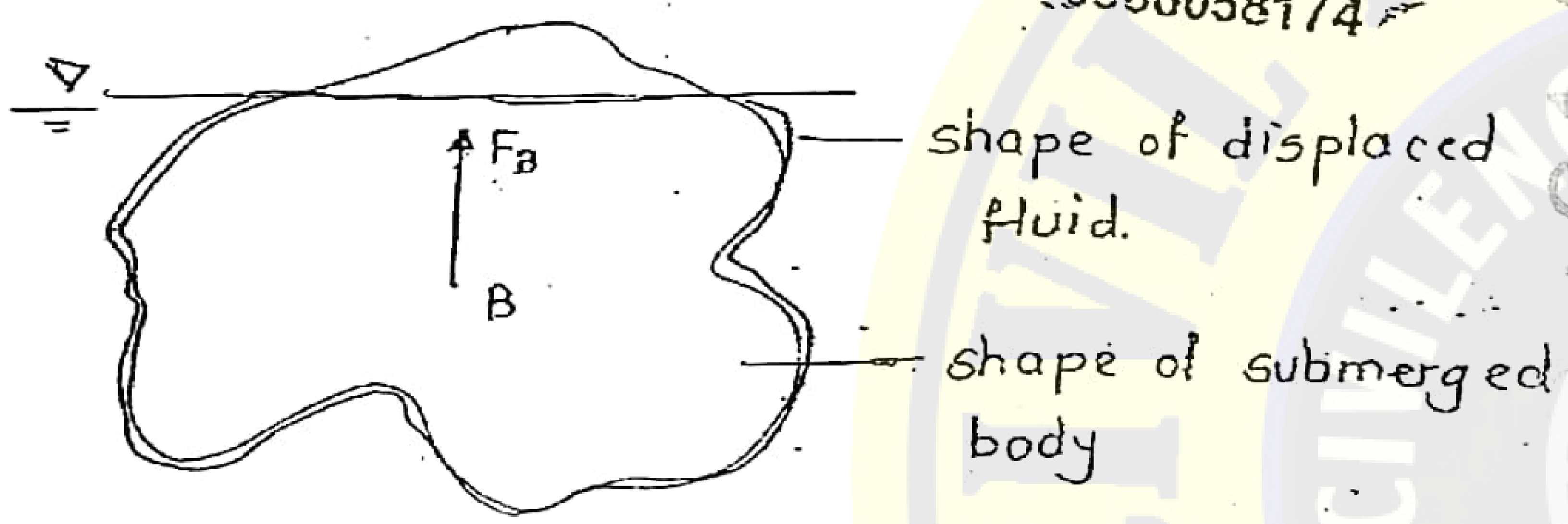
$$\boxed{F_B = \frac{m' g}{(R.D.)}} \qquad \left(R.D. = \frac{\rho_s}{\rho}\right)$$

where R.D. - relative density of body w.r.t. fluid.

Centre of Buoyancy (B):

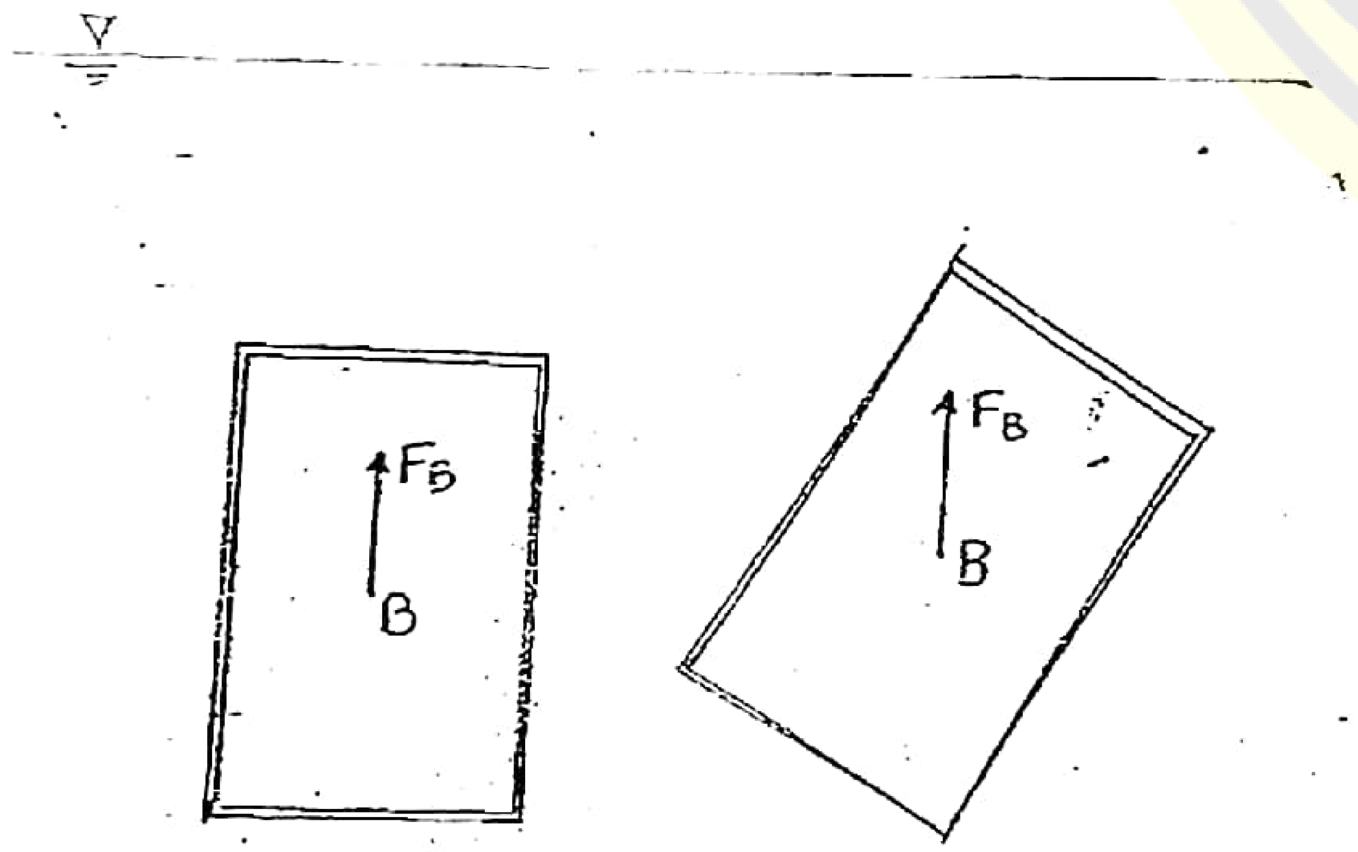
"It is a point in the body from where the buoyant force (upthrust - hydrostatic force) acts."

This point is same as the centre of gravity of displaced fluid. Body can have many surface and thus many centres of pressure. Therefore centre of Buoyancy represents combined effect of all centre of pressures of all surfaces of body.



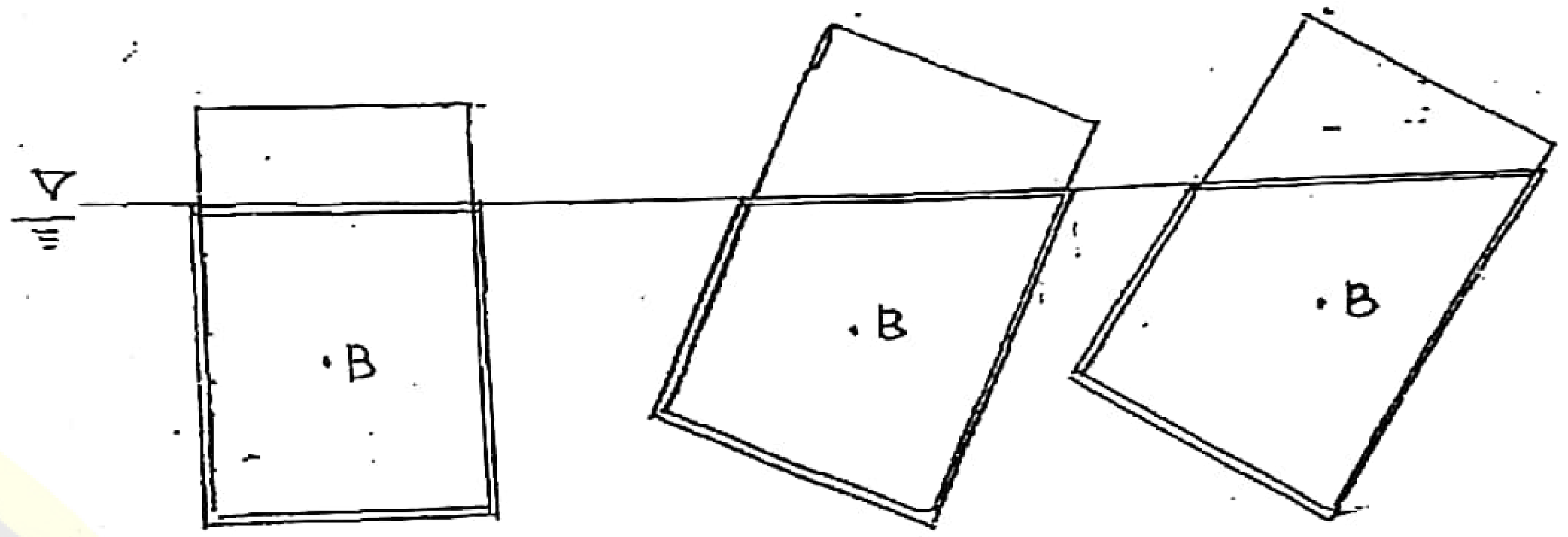
Submerged body:

Under disturbance centre of buoyancy doesn't change.

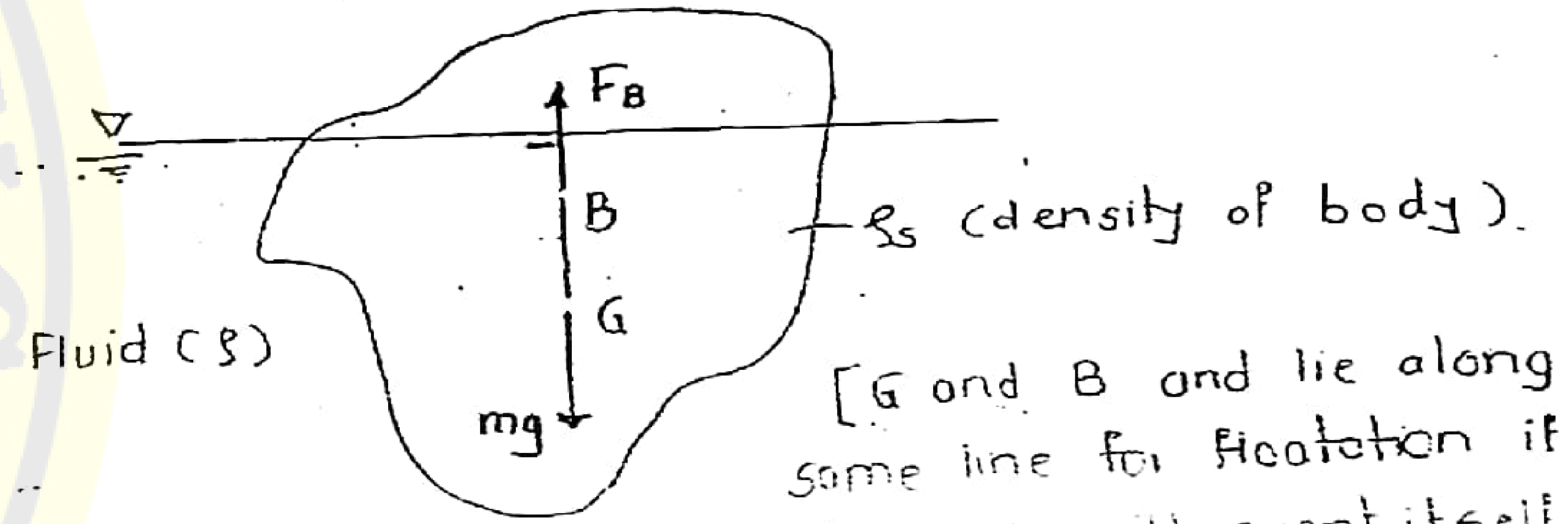


Floating body:

Under disturbance centre of buoyancy changes



Concept of floatation:



[G and B and lie along same line for floatation if n... the body will orient itself bring them in line]

For floatation.

$$mg = F_B$$

$$mg = \frac{m'g}{R.D.}$$

$$m' = m \cdot R.D$$

$$m' \leq m$$

$$m \geq m \cdot R.D.$$

$$R.D. \leq 1$$

$$\left(\frac{\rho_s}{\rho}\right) \leq 1$$

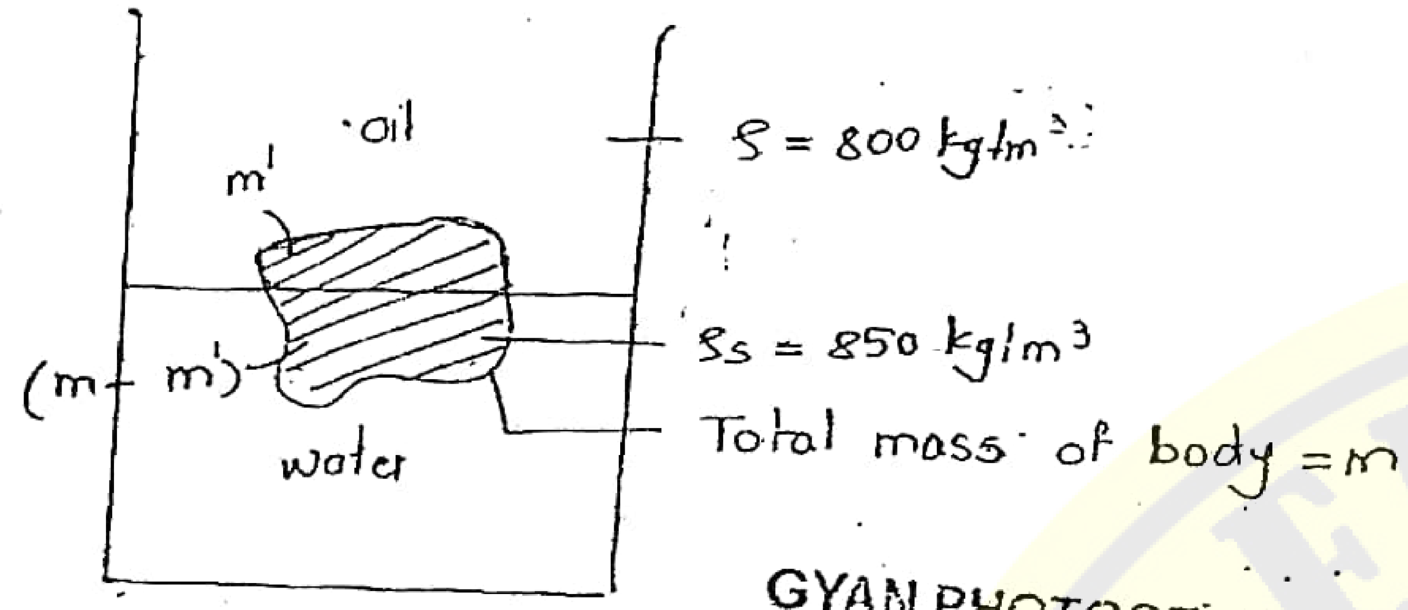
$$\rho_s \leq \rho$$

m - mass of bo...

m' - submerged ... of body

Q. Find fraction of body in oil and water.

2 Marks



For floatation.

$$mg = F_{B\,oil} + F_{B\,water}$$

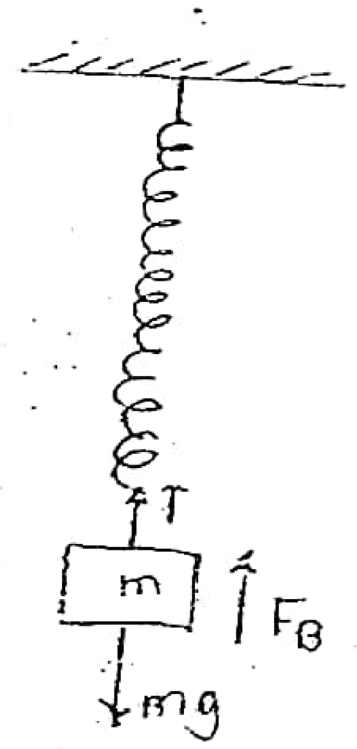
$$mg = \frac{m'g}{\left(\frac{850}{800}\right)} + \frac{(m-m')\cdot g}{\left(\frac{850}{1000}\right)}$$

$$850\ m = 800\ m' + 1000\ m - 1000\ m'$$

$$\frac{m'}{m} = 0.75 \qquad \text{i.e } 75\%\ \text{mass in oil.}$$

Concept of apparent weight :

For real weight (taken in air)



spring balance

T- tension in the spring (reading of spring balance)

F_B - buoyant force by air

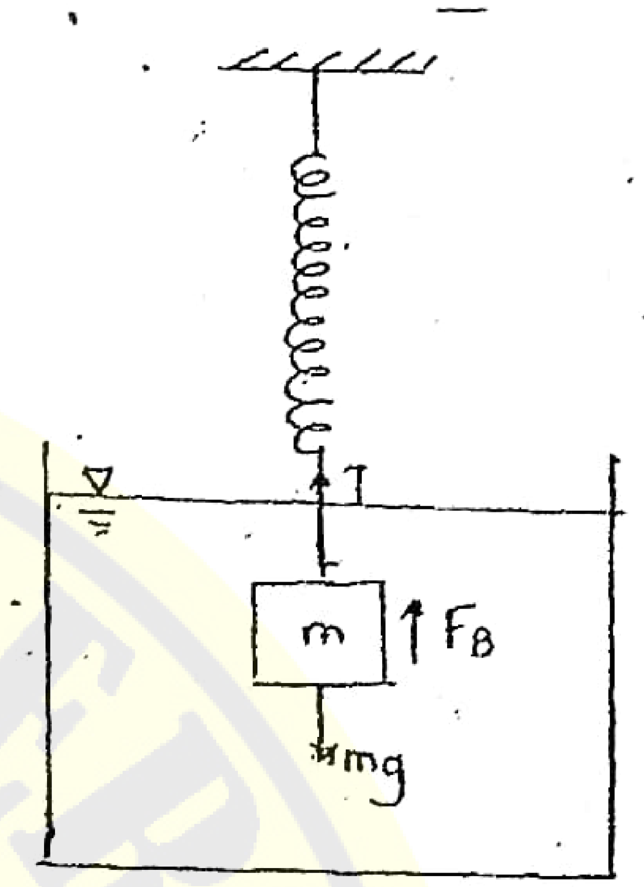
$$T + F_{B\,(by\ air)} = mg$$

$$T = mg - F_{B\,(by\ air)}$$

$$T = mg. \quad - \text{(Real weight)}$$

$F_{B\,(by\ air)} = 0 \quad \because \rho_{air}$ is 0

When weight is taken in liquid



$\rho_s > \rho$ - to submerge the body in fluid.

$$T + F_B = mg$$

$$T = mg - F_B$$

Apparent weight $(T) = mg - F_B$

$\therefore$ Reduction in weight appeared

$$= mg - (mg - F_B)$$

$$= F_B \quad \text{i.e Buoyant force.}$$

Q. A body appears to have 1000 N in water and 1200 N in oil of density 800 kg/m³. find

(i) Real wt. of body

(ii) Mass of body

(iii) Volume of body

(iv) Density of body.

For water.

$$1000\ N = mg - \forall_{body} \times 1000 \times g$$

$$1200\ N = mg - \forall_{body} \times 800 \times g$$

$$1000 + \forall_{body}\ 1000 \times g = 1200 + \forall \times 800 \times g$$

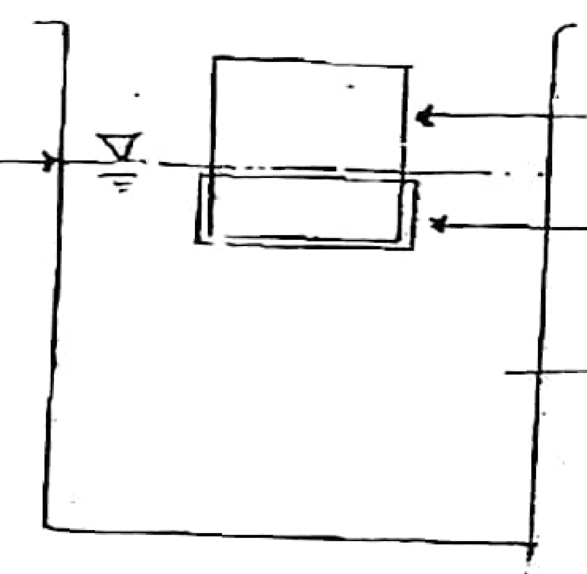
$$1000 + 9000\ \forall_{body} = 1200 + 7200\ \forall_{body}$$

$$1800\ \forall_{body} = 200.$$

$$\forall_{body} = \frac{1}{9}.$$

$$mg = 1000 + 1000 \times 9.8$$

initial level of water when ice cube was floating



For floatation,

$$mg = F_B$$

$$mg = \forall \cdot \rho g$$

$$\therefore \forall = \frac{m}{\rho}$$

Recovery of water. (conservation of mass)

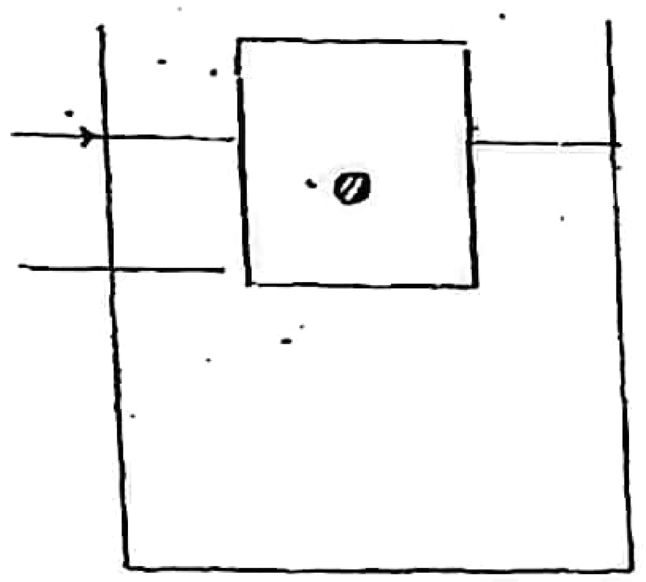
m kg of ice → m kg of water (after melting)

$$\forall_w = \frac{\text{mass of water recovered}}{\text{density of water}}$$

$$\forall_w = \frac{m}{\rho}$$

$$\forall = \forall_{water}$$

The water level will remain same after melting of ice.



$$mg = F_B$$

$$(m_1+m_2)\,g = \forall \cdot \rho \cdot g$$

$$\forall = \frac{m_1}{\rho} + \frac{m_2}{\rho}$$

Recovery of water.

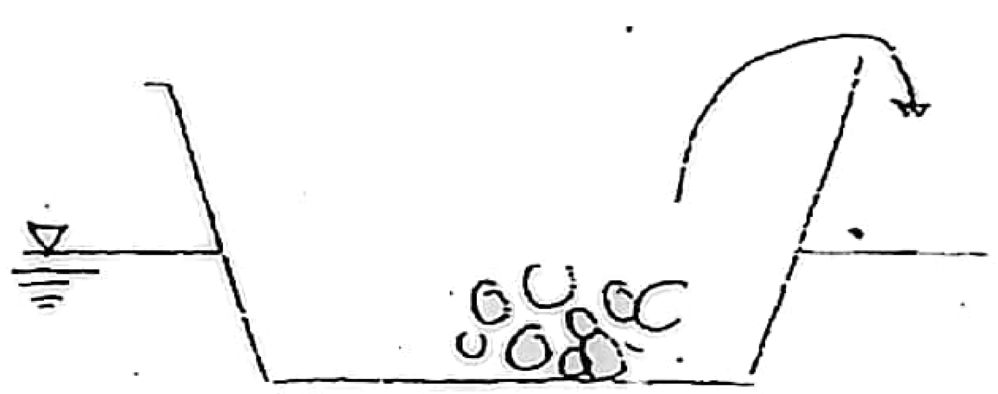
by ice $= \frac{m_1}{\rho}$

by nail $= \frac{m_2}{\rho_{nail}}$

$$\forall_{water} = \frac{m_1}{\rho} + \frac{m_2}{\rho_{nail}}$$

$$\rho < \rho_{nail}$$

The volume of water recovered by nail will be less i.e. water level will go down.



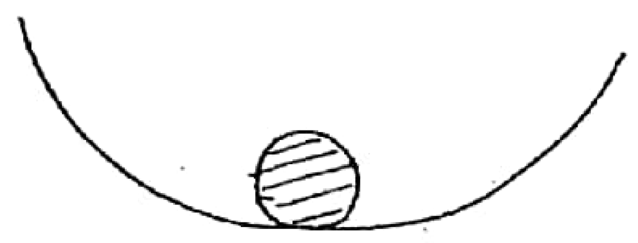
If stone is thrown out of boat then the boat will come up.

If sea water comes from sea water to river water, the

Equilibriums and types of Equilibriums :

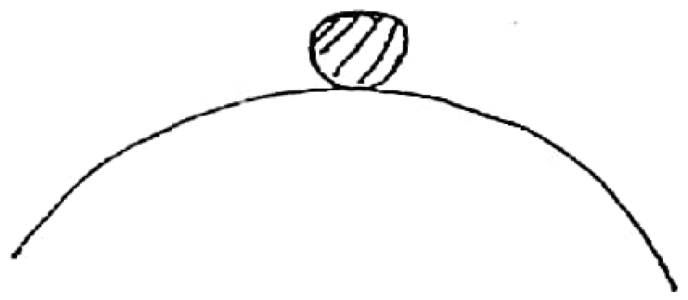
$\Sigma \vec{F} = 0$ - translatory equilibrium

$\Sigma \vec{M} = 0$ - rotationary equilibrium.



stable equilibrium

(body will not leave its original equilibrium position after applied force)

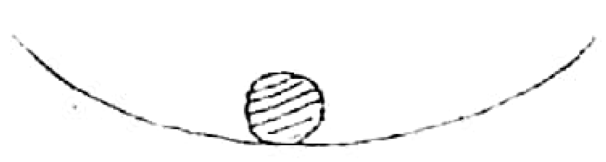


unstable equilibrium

(body will never come back to its original equilibrium position)

Neutral equilibrium

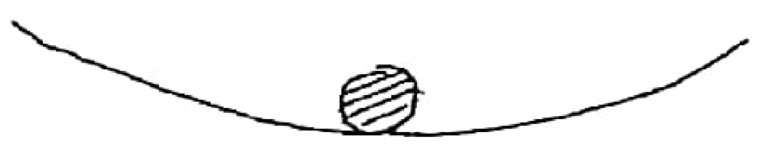
(body will gain the new equilibrium position)



→ More stable

[The body will oscillate (the no. of oscillation depends on the fluid properties)

More stability - less period of oscillation.]

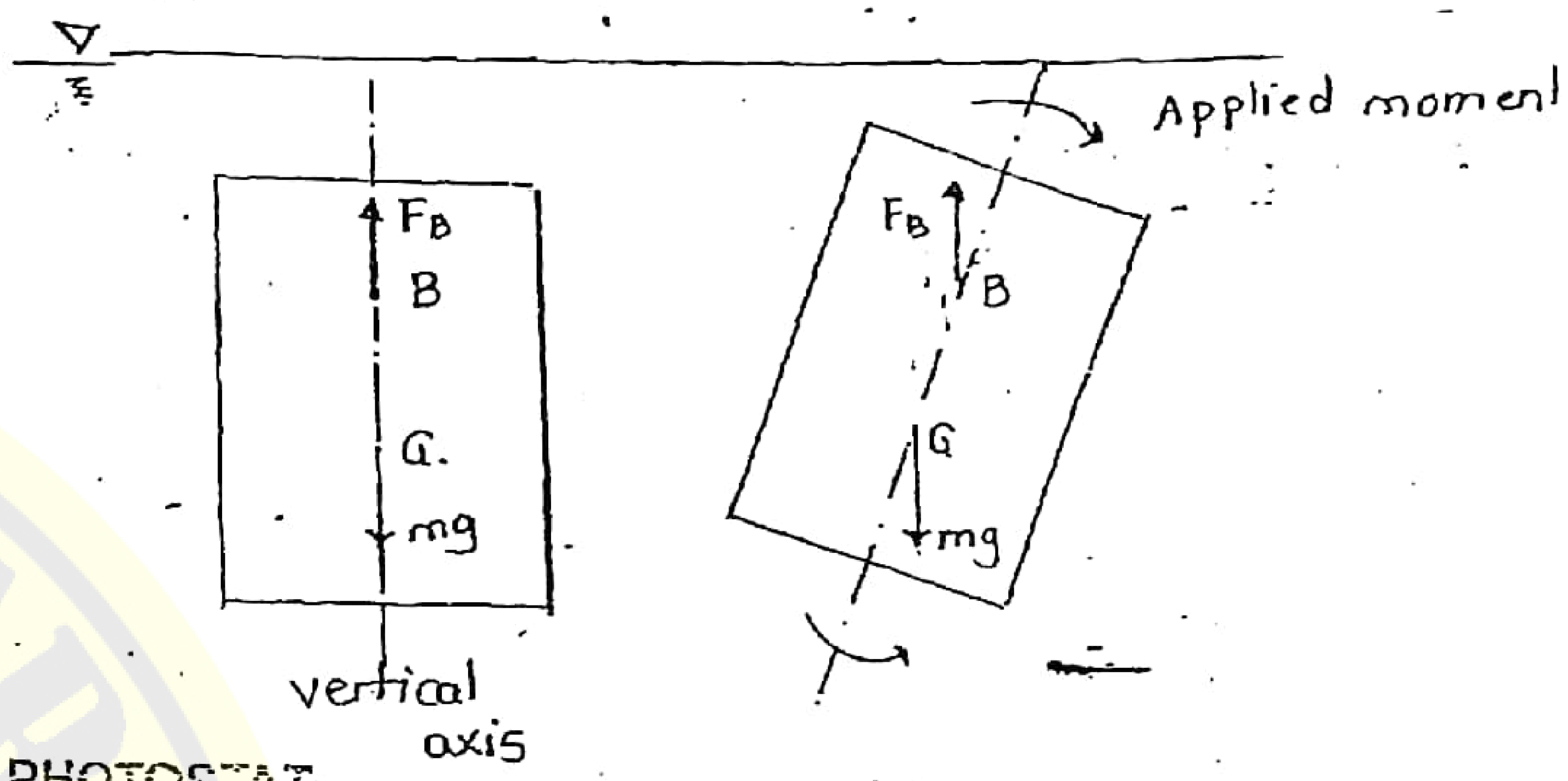


→ less stable

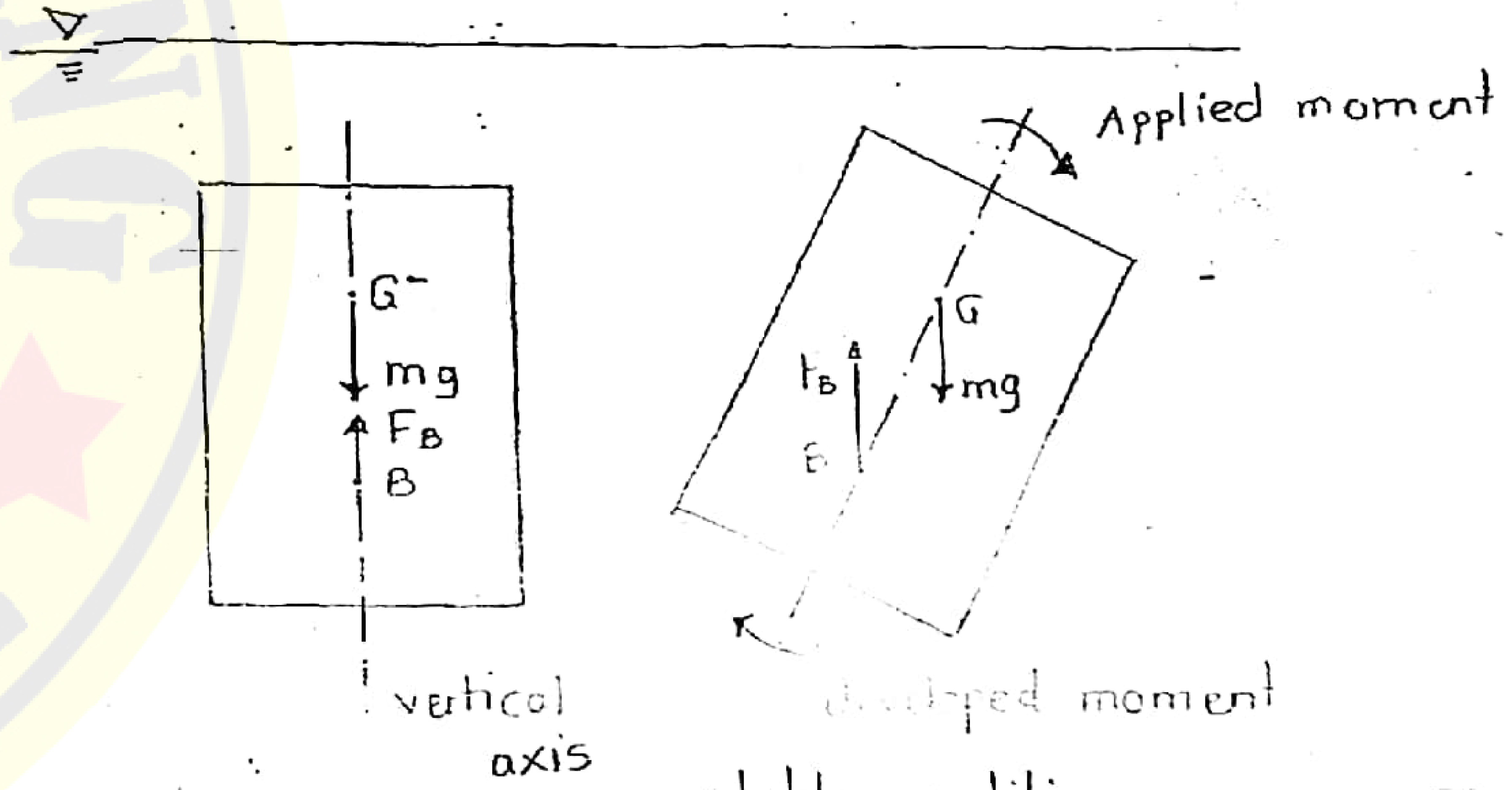
→ stability zero (neutral equilibrium)

The neutral equilibrium is obtained

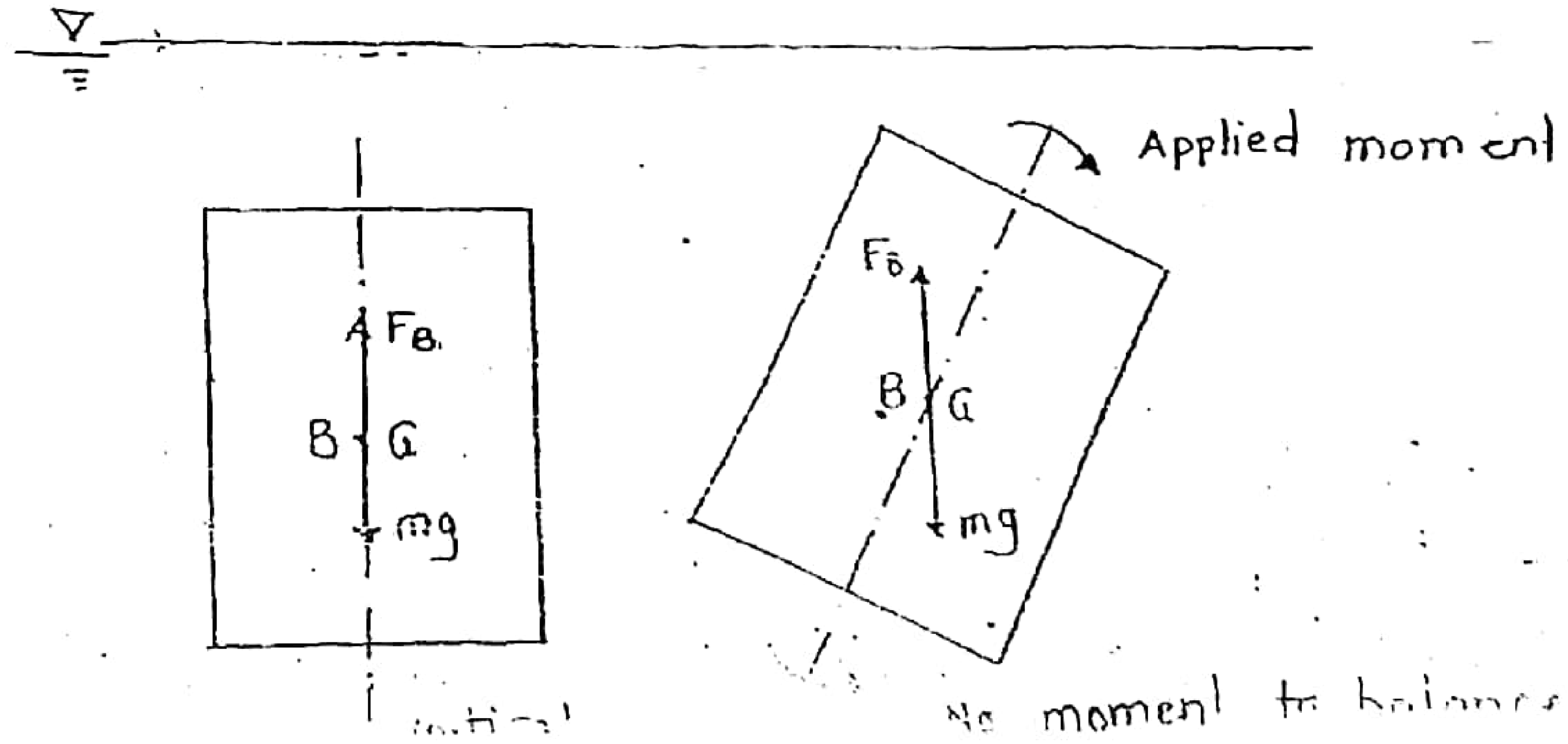
Stability of submerged bodies :



Stable condition



Unstable condition



No moment to balance

Stability of floating bodies:

When the floating bodies are disturbed angularly, there centre of buoyancy is continuously changing. Therefore the stabilities of floating bodies are analysed w.r.t. the different point known as Metacentric point which is also known as Metacentre. It is represented by M.

Metacentre (M):

"It is defined as point of ~~intersection~~ intersection of buoyant force line with the vertical axis of the body under very small angular tilt given to the body.

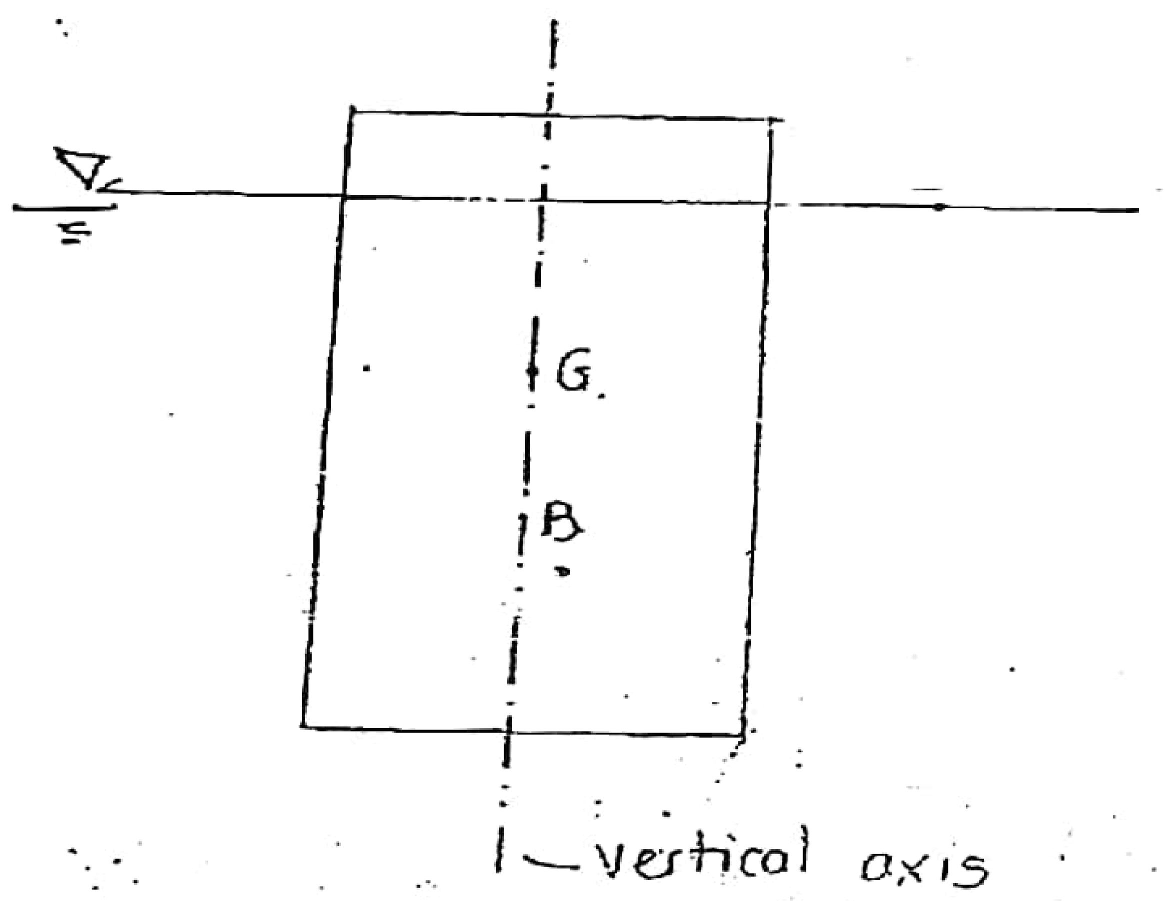
The disturbance should be small so as to not change the position of body.

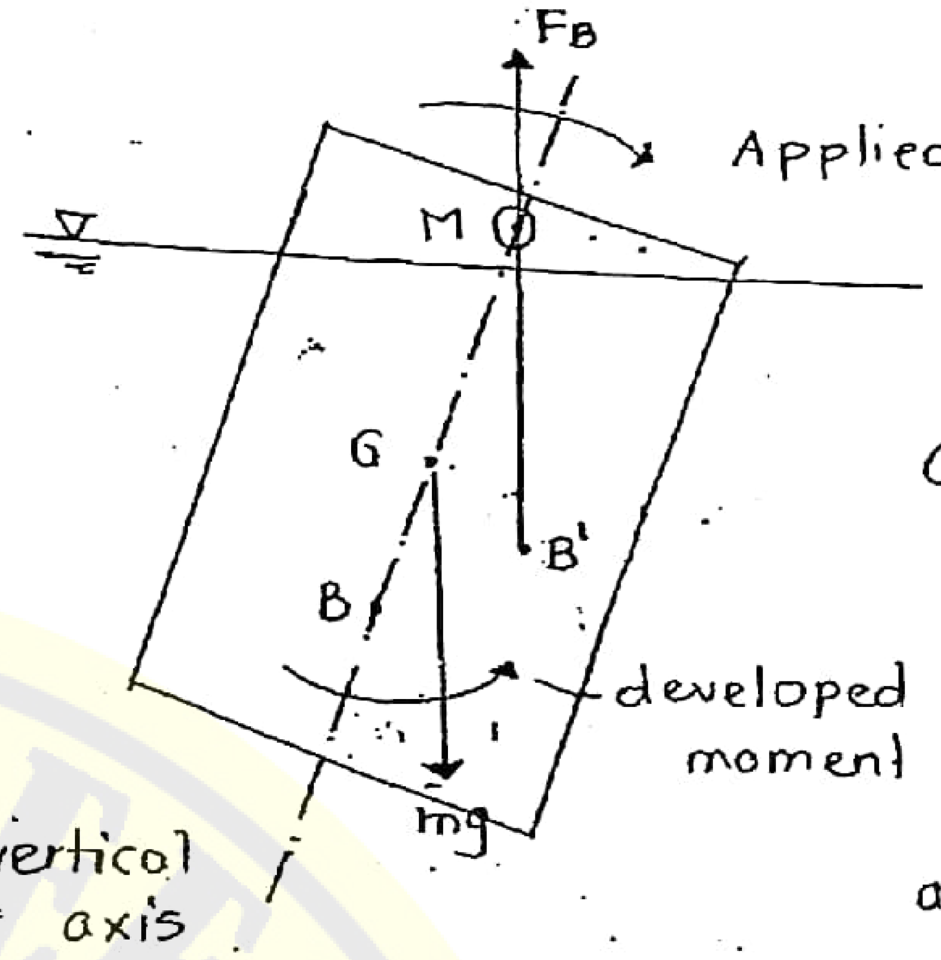
$\sin 2^\circ = 0.0348$

$2^\circ = 0.349^c$ i.e. θ is very small

$\sin \theta \simeq \theta$

This point may be inside or outside the body. It is also defined as a point about which the body is oscillating when they are slightly disturbed and released.

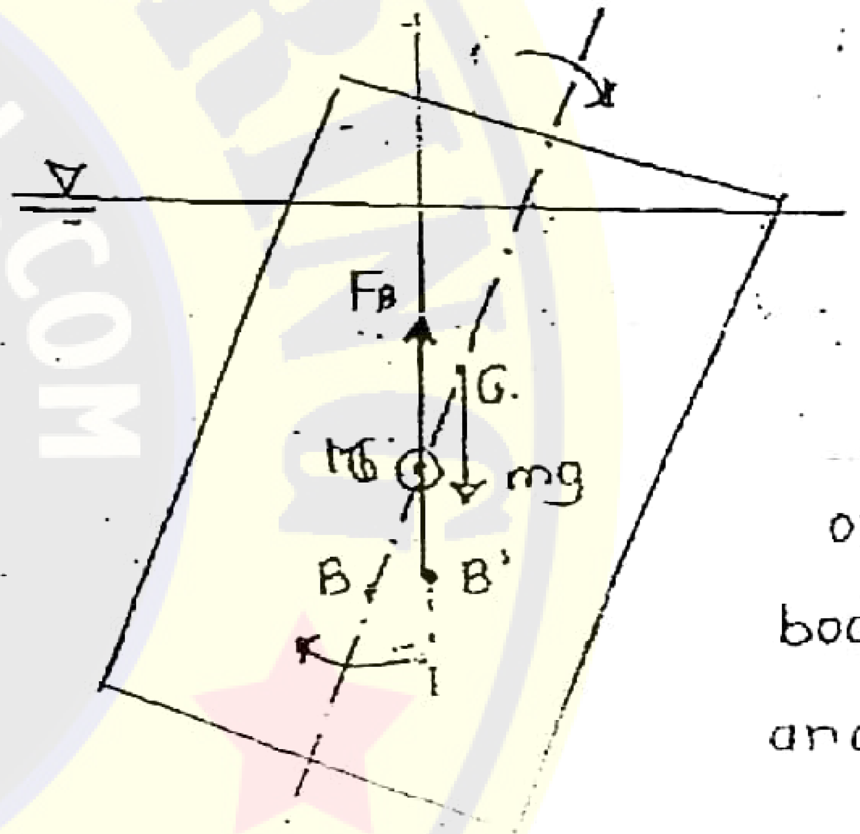




stable equilibrium

(As the applied moment is balanced by developed moment due to mg about M)

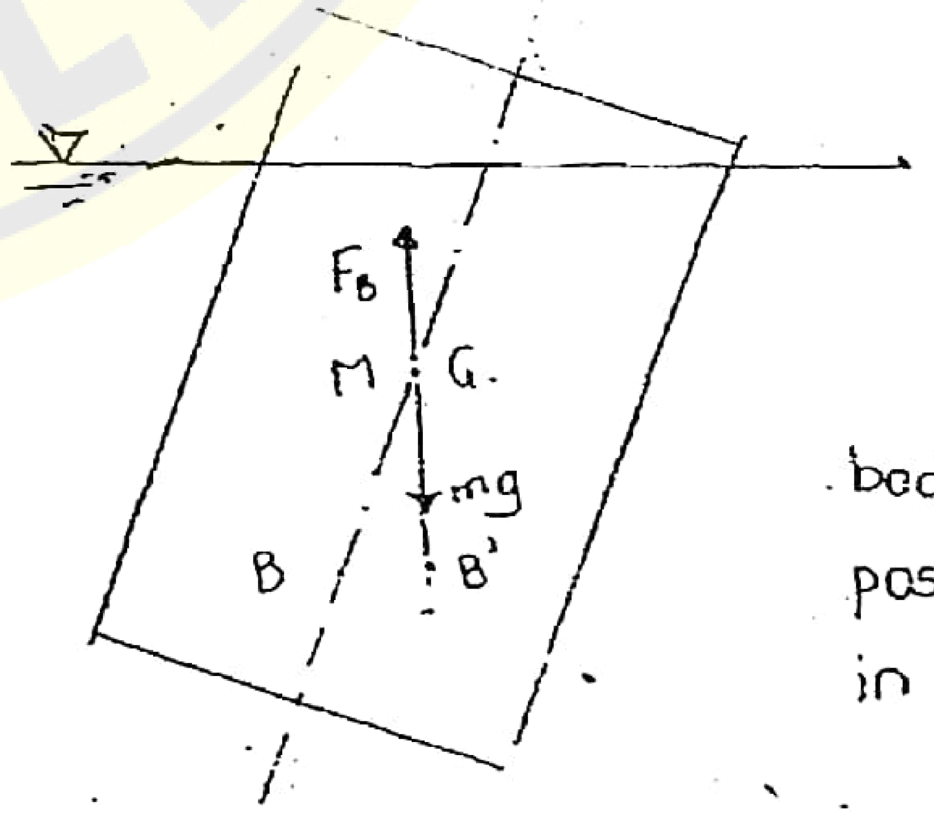
∴ Metacentre (M) should lie above the centre of gravity (C.G.)



Unstable equilibrium

(applied moment causes development of moment in same direction i.e. body will not come to original position or another equilibrium

Metacentre (M) should lie below C.G.

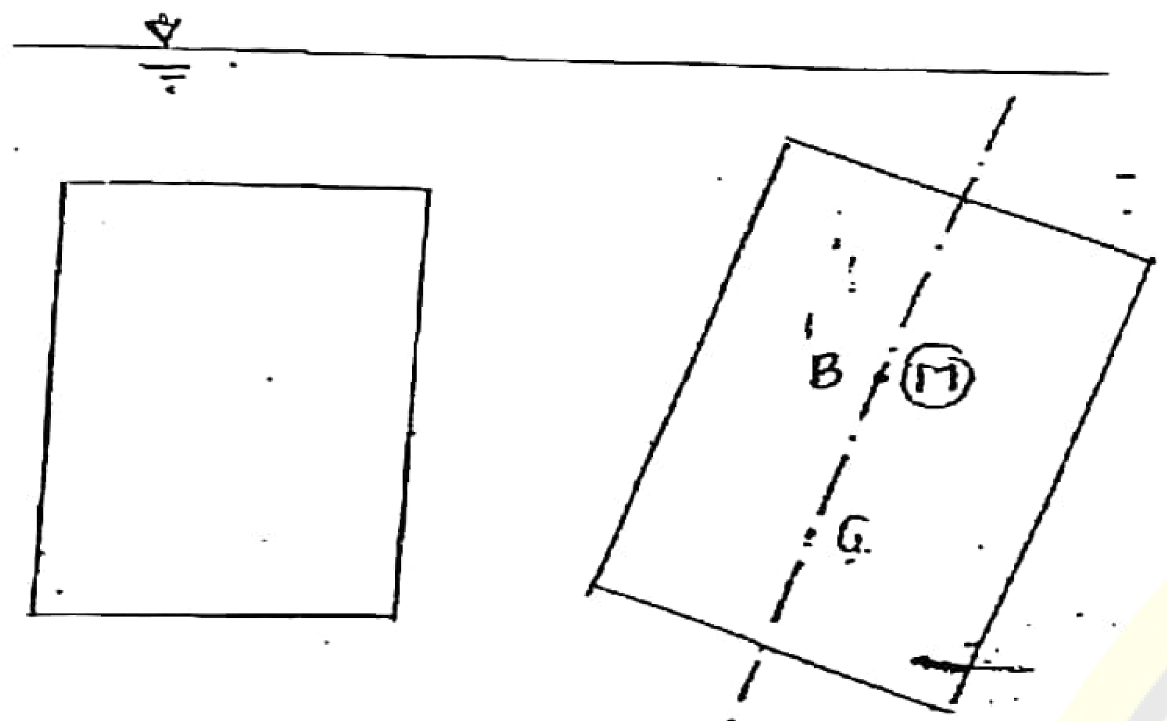


Neutral Equilibrium

(Applied moment rotates the body where it attains new equilibrium position i.e. no moment is developed in the body.

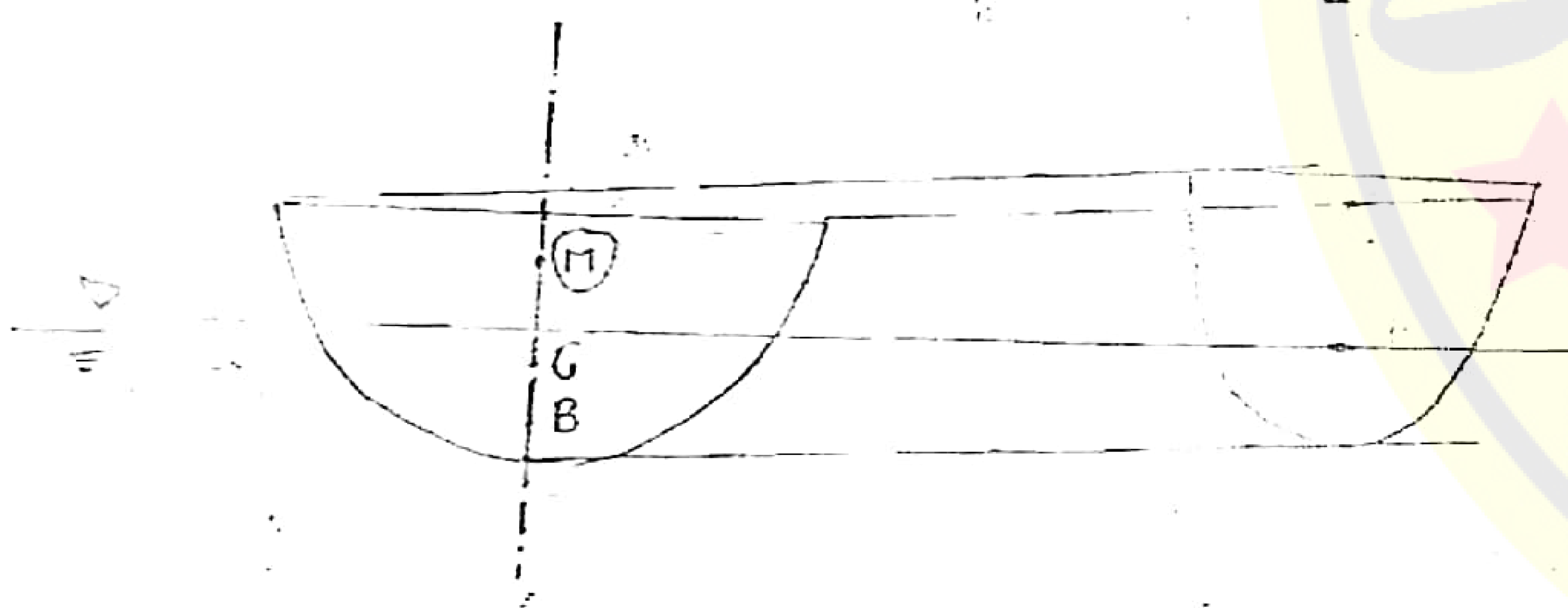
Metacentre coincides with G.

For submerged body.



In a submerged bodies the centre of buoyancy is Mentacentre itself. i.e. the original criteria or the point of reference for checking the stability of a body is Metacentre always.

Metacentric height :



The vertical distance between the M and G is known as Metacentric height. (GM)

$$GM = BM - BG$$

$$GM = \frac{I}{\forall} - BG$$

If $GM > 0$ - stable equilibrium (M above G)

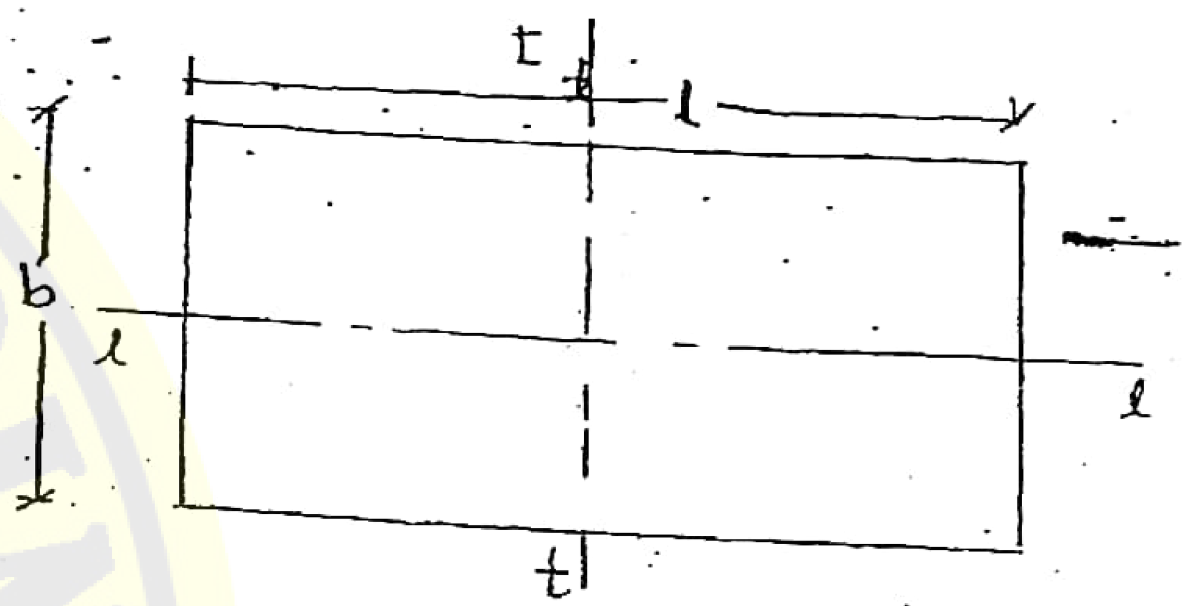
$GM < 0$ - unstable equilibrium (M lies below G)

$GM = 0$ - neutral equilibrium.

$\forall$ - volume of fluid displaced (volume of submerged mass of body)

I - Moment of inertia of surface of body which is intersected by the free water surface.

Face intersected by free surface:



If the body oscillates about the longitudinal axis, the phenomenon is called Rolling phenomenon.

$$I_{ll} = \frac{lb^3}{12} \quad (\text{@ longitudinal axis})$$

If the body oscillates about the transverse axis, the phenomenon is called Pitching phenomenon.

$$I_{tt} = \frac{bl^3}{12} \quad (\text{@ transverse axis})$$

As $I_{ll} <<< I_{tt}$

The body oscillations will be more about the longitudinal axis. (less stable about longitudinal axis)

$$(GM)_{Rolling} <<< (GM)_{pitching}$$

the body will be sensitive for rolling action i.e. the rolling is more dangerous

The stabilities of floating bodies are always designed under the rolling

$$GM = \frac{I_{ll}}{\forall} - BG.$$

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{k^2}{(GM)\cdot g}}$$

k - radius of gyration.

$I = A\cdot k^2$

$$T \propto \frac{1}{\sqrt{GM}}$$

i.e. More is the metacentric height less is the period of oscillation.

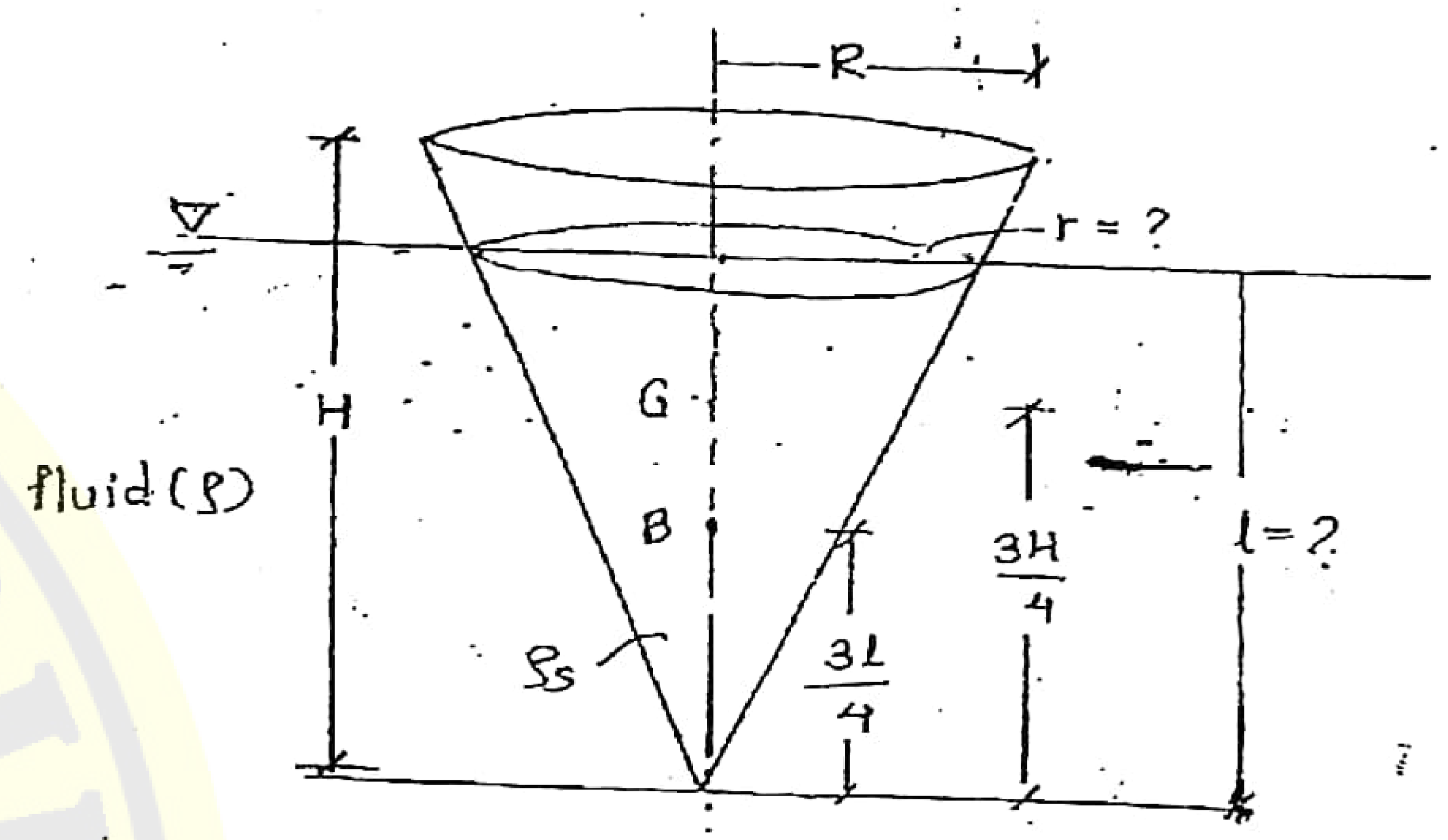
Passenger ships (Cargo ships) have metacentric height 0.5-1 m for the purpose of more comfort to the passengers. The less metacentric height indicates less stable and high period of oscillation i.e. deflected ship takes more time to reach its original position so smoothly that passengers inside will barely notice.

For War ships metacentric height is more (1-1.8 m)

War ships are meant to work in ~~battle~~ situations where planes, helicopters may land on it. The landing of aircraft is critical phenomenon which needs to be taken care. Thus the War ship should stable so that period of oscillation is very less when striked by the water waves. Thus Metacentric height is large.

Q. A cone having a max. radius R, height H and density ρ_s is floating in fluid of density ρ with its axis vertical and apex down. Find the condition for the stability of cone.

0 Marks



Relative density (R.D.) $= \frac{\rho_s}{\rho}$

For floatation.

$$mg = F_B$$

$$g\left(\frac{1}{3}\pi R^2 H\right)\rho_s = \forall\,\rho g$$

$$\frac{1}{3}\pi R^2 H\cdot\rho_s = \frac{1}{3}\pi r^2 l\cdot\rho$$

$$r^2 l = R^2 H\cdot\frac{\rho_s}{\rho}$$

$$r^2 l = R^2 H\ (R.D.)$$

$$\frac{R}{H} = \frac{r}{l}$$

$$r = \left(\frac{R}{H}\right)l \qquad \text{from similar triangl}$$

$$\left(\frac{R^2}{H^2}\right)l^2\cdot l = R^2\cdot H\ (R.D.)$$

$$l^3 = H^3\cdot(R.D.)$$

$$l = H\ (R.D.)^{1/3}$$

$$r = \frac{R}{H} \cdot H \cdot (R.D.)^{1/3}$$

$$= R \cdot (R.D.)^{1/3}$$

To find BG.

$$BG = \frac{3H}{4} - \frac{3l}{4}$$

$$= \frac{3}{4}(H - l)$$

$$= \frac{3}{4}\left(H - H \cdot R.D.^{1/3}\right)$$

$$BG = \frac{3}{4} H \left(1 - R.D.^{1/3}\right)$$

To find BM.

$$BM = \frac{I}{\forall}$$

$$= \frac{\frac{\pi r^4}{4}}{\frac{1}{3}\pi r^2 \cdot l}$$

$$= \frac{3}{4} \frac{r^2}{l}$$

$$= \frac{3}{4} \frac{R^2 (R.D.)^{2/3}}{H (R.D.)^{1/3}}$$

$$B.M. = \frac{3}{4} \frac{R^2}{H} (R.D.)^{1/3}$$

$$GM = B.M - B.G.$$

for stability.

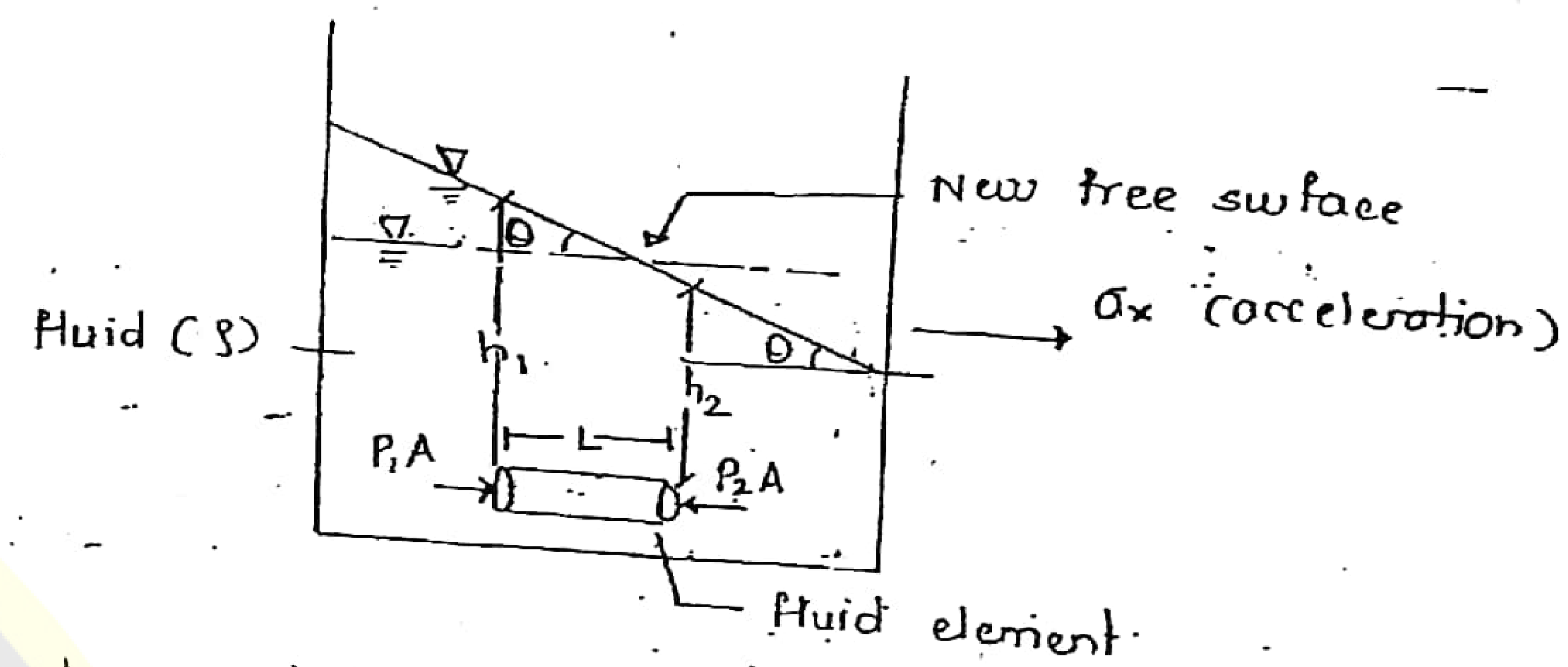
$$GM > 0$$

$$BM > B.G.$$

$$\frac{3}{4} \frac{R^2}{H} (R.D.)^{1/3} > \frac{3}{4} H \left(1 - R.D.^{1/3}\right)$$

$$\frac{R^2}{H^2} > \frac{1 - R.D.^{1/3}}{R.D^{1/3}}$$

Concept of linearly accelerated vessels containing liquid:



By Newton's 2nd law of motion.

$$P_1 A - P_2 \cdot A = m \cdot a_x$$

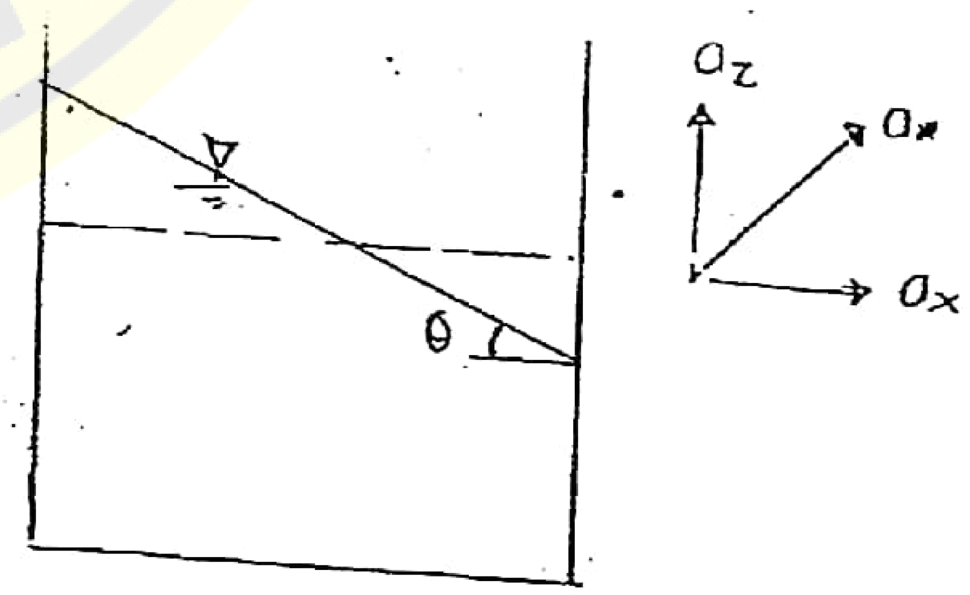
$$(P_1 - P_2) \cdot A = (A \cdot L \cdot \rho) \cdot a_x$$

$$(P_1 - P_2) = L \cdot \rho \cdot a_x$$

$$\rho g (h_1 - h_2) = L \rho \cdot a_x$$

$$\left(\frac{h_1 - h_2}{L}\right) = \frac{a_x}{g}$$

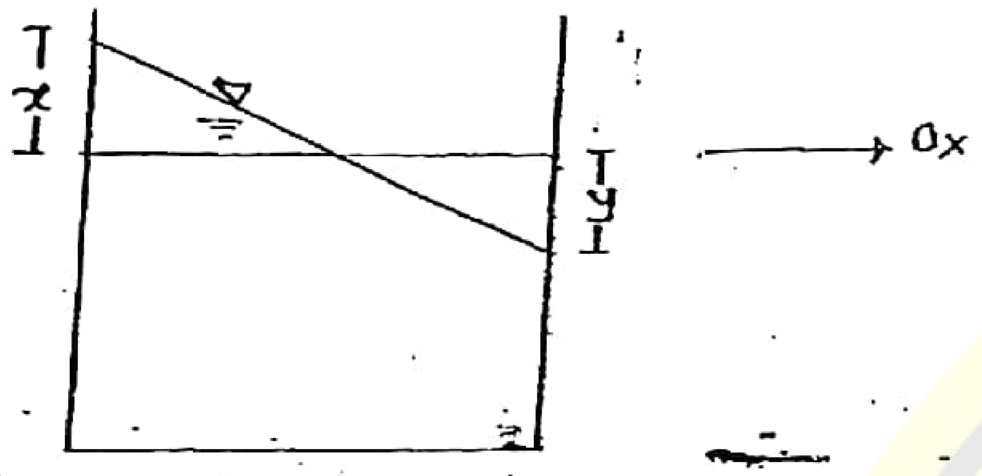
$$\tan\theta = \frac{a_x}{g}$$



$$\tan\theta = \frac{a_x}{g + a_z}$$

Conservation of volume :

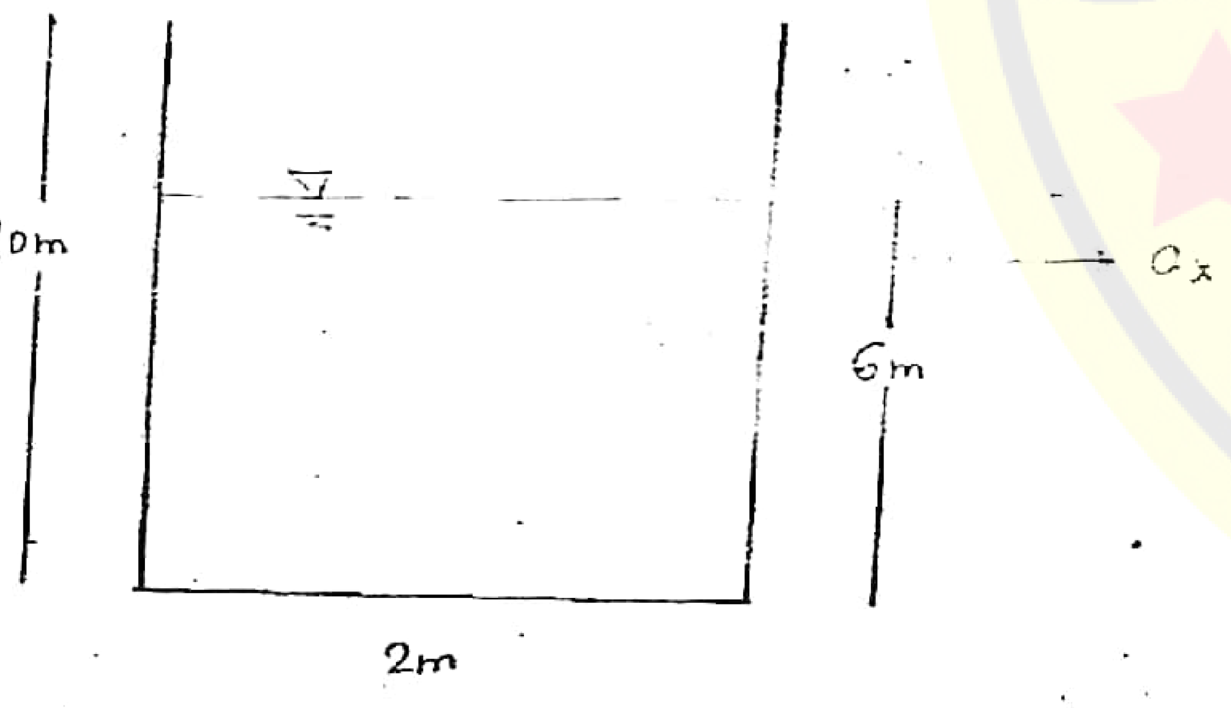
It is applicable if not even a single drop of liquid is spilled out.



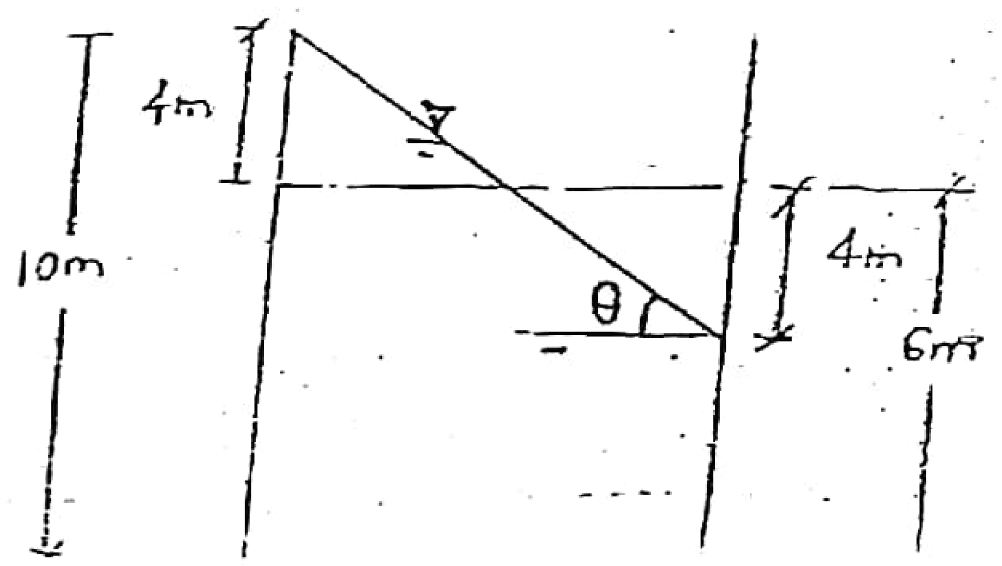
If a container filled with liquid is accelerated in horizontal direction then rise of liquid level on one side is equal to fall of liquid level on other side.

$$x = y.$$

Q. Find the max. acceleration so that water doesn't spill out.



Width = 3m



As water is not spilling out, law of conservation of volume is applicable.

$$\tan\theta = \frac{8}{2} = 4.$$

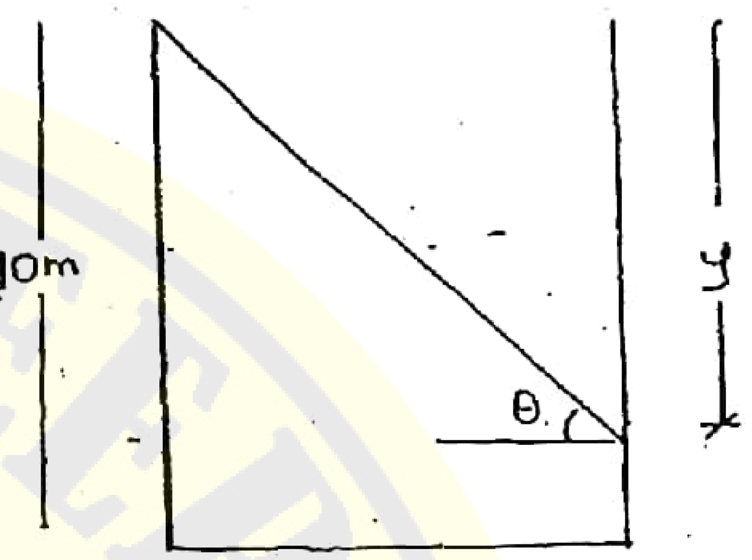
$$\tan\theta = \frac{(a_x)_{max}}{g}$$

$$(a_x)_{max} = 4.9$$

Q. Find the volume of water spilled if $a_x = 45\ m/s^2$

$$\tan\theta = \frac{a_x}{g} = \frac{45}{9.81}$$

$$\tan\theta = 4.587 \quad < 5 \qquad \text{For } \left(\frac{10}{2} = \tan\theta\right)$$



$$\tan\theta = \frac{y}{2}$$

$$y = 2\times 4.587$$

$$= 9.174\ m$$

$$V_{final} = \left[\frac{9.174\times 2}{2} + 0.826\times 2\right]\times 3$$

$$V_{initial} = 2\times 6\times 3$$

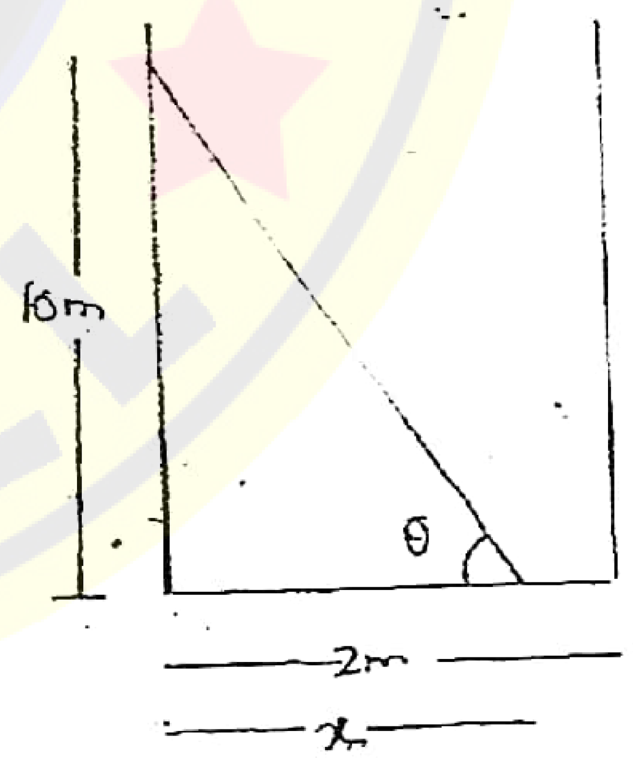
$$V_{spilled} = 3.522\ m^3$$

Q. Find spilled volume if $a_x = 60\ m/s^2$

$$\tan\theta = \frac{60}{9.81} = 6.116$$

$$\tan\theta = \frac{10}{x}$$

$$x = 1.635\ m$$



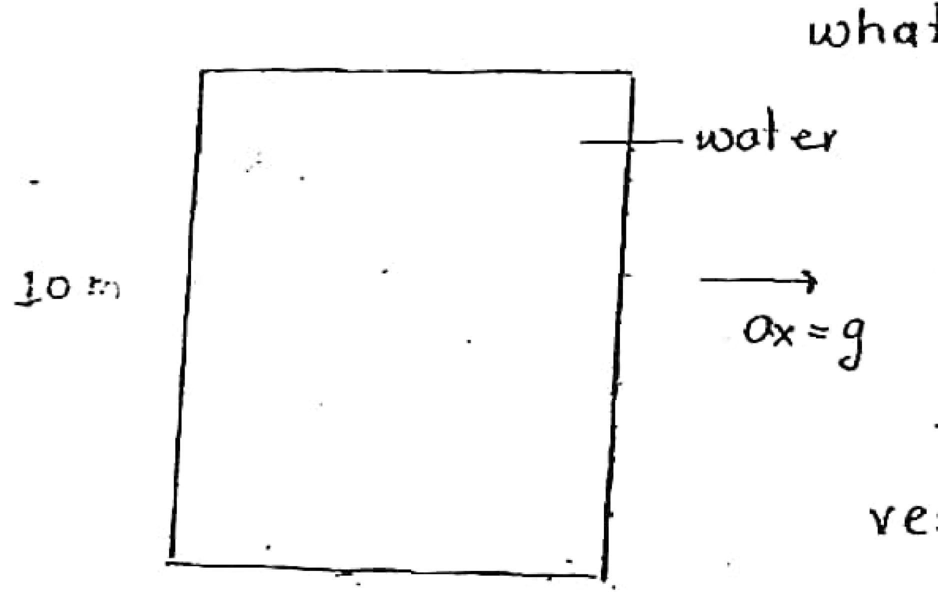
$$V_{final} = \left(\frac{10\times 1.635}{2}\right)\times 3$$

$$=$$

$$V_{initial} = (2\times 6\times 3)$$

$$V_{spilled} = 11.475\ m^3$$

Q. If the closed container moved with an acceleration g m/s². what is force at the bottom?



width = 3m

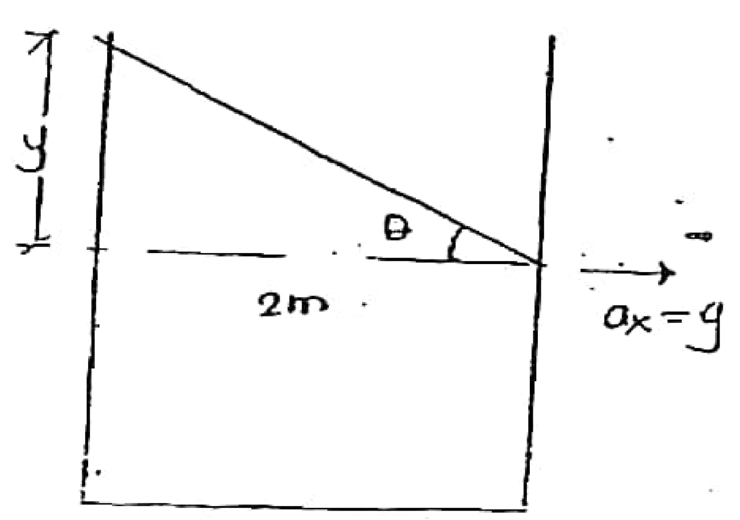
The hydrostatic force will always there in the fluid.

∴ Hydrostatic force at the bottom of vessel

$F_1 = (\rho g h) \cdot A$

$= (1000 \times 9.8 \times 10) \times (2 \times 3)$

=



If the container was open, the volume of water spilled $\forall$.

As the container is closed, $\forall$ water will apply force on top surface of container as it is trying to spill.

$\tan\theta = \frac{a_x}{g}$

$\frac{y}{2} = \frac{g}{g}$

$y = 2$

$\forall_{spilled} = \left(\frac{1}{2} \times 2 \times 2\right) \times 3$

$= 6\ m^3$

Force applied on the top (or the force applied on bottom due to $\forall_{spilled}$)

$F_2 = \rho g \forall$

$= 1000 \times 9.8 \times 6$

=

Total force on bottom = $F_1 + F_2$

KINEMATICS OF FLUID FLOW

most difficult

It is the study of fluid particle motion without the help of basic cause of motion i.e. force.

1. Lagrangian method : (used for Ph.D)

This approach is particle concentration approach i.e. the entire concentration goes to a particular fluid particle, and its motion is analysed. After completing the study of motion of one fluid particle, the concentration goes to another fluid particle. Since the number of fluid particles in a system are very large, therefore this approach is highly time consuming but very accurate approach. Just because of time consumption to be high, this approach is not used in Classical Fluid Mechanics.

computational fluid dynamics + Research finite volume / element Technique (5-6 yrs

2. Eulerian method :

(1st to read/observe vibrations of cantilever beam) Father → Navier Stokes partial fluid eqn (M.S. project)

This method is space concentration approach i.e. the entire concentration goes to a particular space or zone and all the fluid particles passing through that space or zone are analysed simultaneously. Therefore, the results obtained with the method are the average results which are not correct particle by particle but on an average these results are 100 % correct for overall bulk motion of fluid particles. Therefore this approach is very time saving approach. Hence we prefer this approach in classical fluid Mechanics.

e.g. If one travells 20 km distance in 15 min ($\frac{1}{4}$ hr) with different velocities.

Avg. velocity = $\frac{20}{(1/4)}$ = 80 km/hr.

i.e. when one travells with constant velocity of 80 km/hr

Different types of flows in the fluid flow system:

1. Steady and unsteady flows:

If the properties in flow are not changing w.r.t time, such a flow is known as steady flow. and if the properties are changing w.r.t. time it is unsteady flow.

$$\left.\frac{\partial R}{\partial t}\right|_{space-fixed} = 0 \qquad R- \text{fluid properties.}$$

2. Uniform flow and non-uniform flow:

If the properties in the flow are not changing w.r.t. space then such a flow is known as uniform flow

$$\left.\frac{\partial R}{\partial space}\right|_{time-fix} = 0 \qquad \text{uniform flow.}$$

3. Incompressible flow and compressible flow:

If the density is not changing w.r.t pressure then such a flow is known as Incompressible flow.

$$\frac{\partial R}{\partial P} = 0 \qquad \text{Incompressible flow.}$$

4. Irrotational flows and rotational flows:

In a flow if the fluid particles are also rotating about there own centre of masses then such a flow is known as rotational flow and if the particles are not rotating about their own centre of masses then such a flow is known as irrotational flow.

e.g. The swirling water in a bucket is rotational flow.

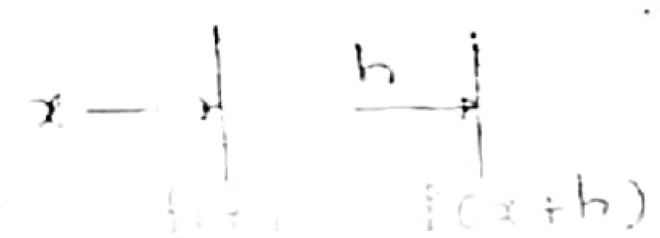
5. Laminar flow and turbulent flow:

"If all the fluid particles lying in a layer are having there velocities in the direction of flow of layers then there will not be any kind of intermixing of the fluid particles between the adjacent layers. Such a well organised flow of fluid particles in the laminated form of layers is known as Laminar flow. This flow is also known as Streamline flow." (Name given by Raynold)

If all the fluid particles lying in a layer are having their velocity component in different directions then there will be huge intermixing of fluid particles between the adjacent layers. Such a most chaotic flow is known as Turbulent flow. Shear force between particles also contribute

Some mathematical tools:

1: Taylor series:



$$f(x+h) = f(x) + f'(x)\,h + f''(x)\frac{h^2}{2!} + f'''(x)\frac{h^3}{3!} + \dots$$

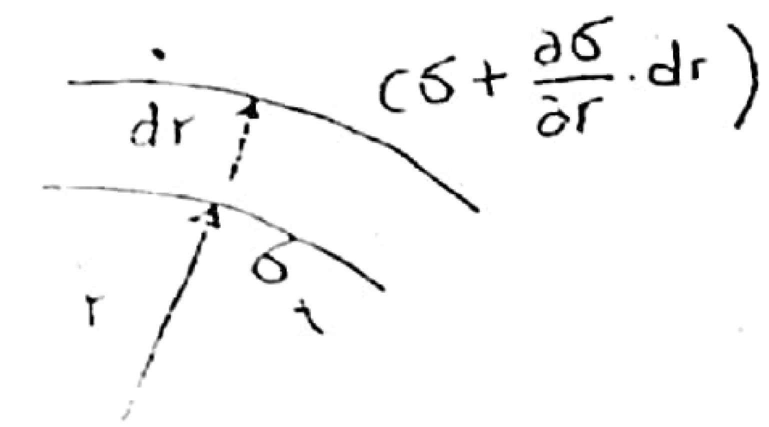
If h is very small i.e. of the order of dx.

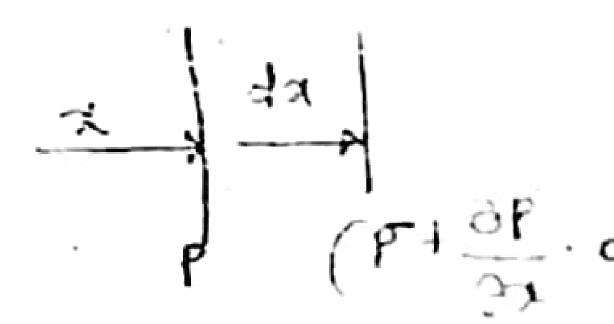
$$f(x+dx) = f(x) + f'(x)\cdot dx$$

– (Taylor series approximation) upto 1st order

Application:

σ-stress at r





2. If F is function of many variables

$$F = f(x, y, z)$$

Total change = sum of partial changes in x, y and z.

$$dF = \underbrace{\frac{\partial F}{\partial x} \cdot dx}_{\text{(Partial change in x- direction)}} + \frac{\partial F}{\partial y} \cdot dy + \frac{\partial F}{\partial z} \cdot dz$$

(gradient) (length in X-direction)

Continuity equation:
(conservation of mass)

In general, for 3-D flows, the velocity is given by

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

where,
u, v, w are velocity components in X, Y and Z directions.

The magnitudes of u, v, w are the functions of (x, y, z, t)

$$\vec{V} = f(x, y, z, t)$$

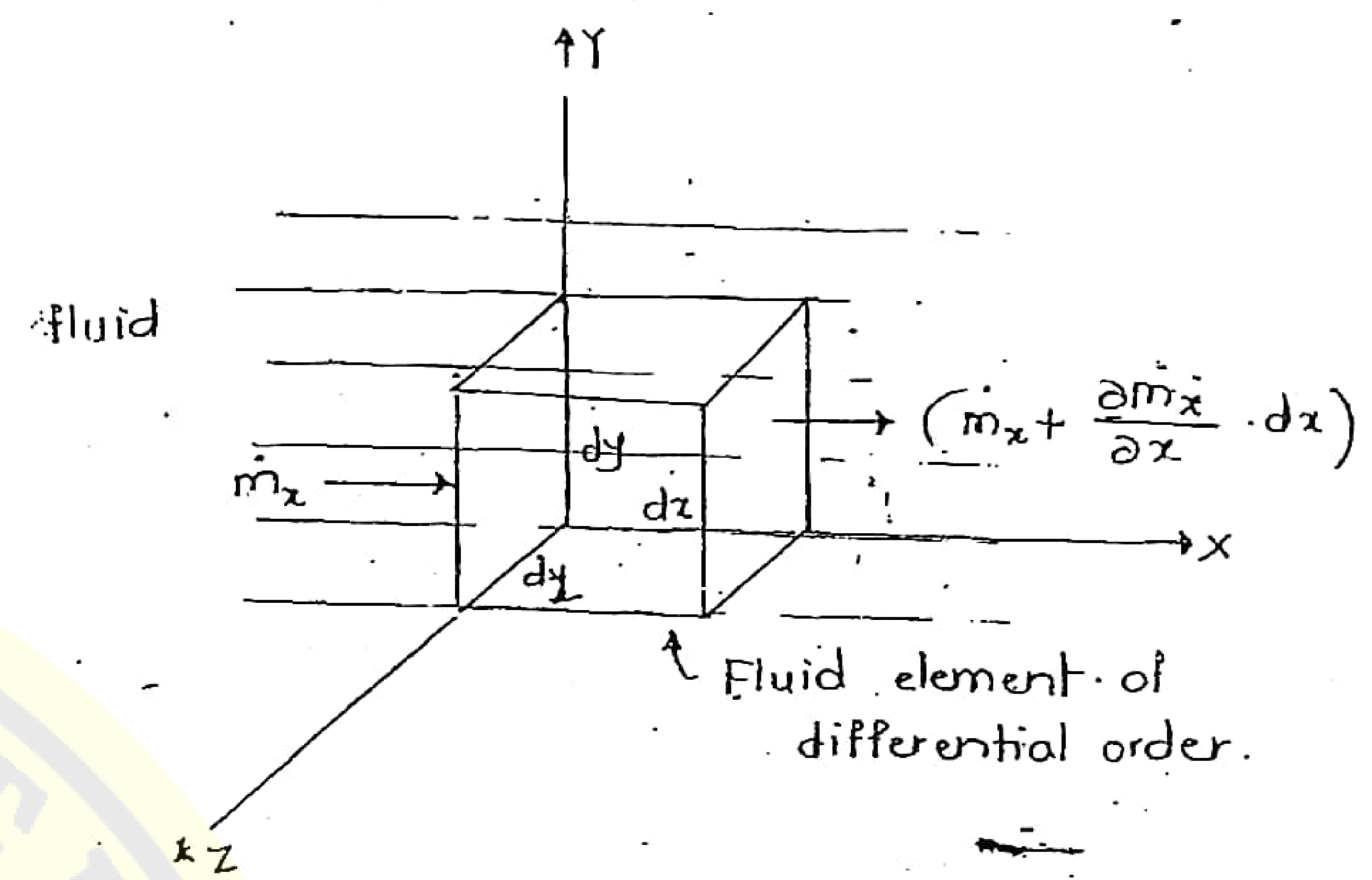
conservation of mass

$$\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{stored}$$

where, $\dot{m}_{in}$ - mass entering the system per sec.
$\dot{m}_{out}$ - mass leaving the system per sec
$\dot{m}_{stored}$ - mass stored per sec.

$\dot{m}_{in}$ and $\dot{m}_{out}$ are surface phenomenon (as the mass will enter or leave the system through a section)

$\dot{m}_{stored}$ is bulk / volumetric phenomenon as mass is



In X- direction,

$$(\dot{m}_{in} - \dot{m}_{out})_x = \dot{m}_x - \left(\frac{\partial \dot{m}_x}{\partial x} \cdot dx + \dot{m}_x\right)$$

$$= -\frac{\partial \dot{m}_x}{\partial x} \cdot dx$$

$$= -\frac{\partial}{\partial x}(\underbrace{\rho \cdot dy \cdot dz \cdot u}_{\text{rate of mass entering per sec}}) \cdot dx$$

$$= -\frac{\partial}{\partial x}(\rho \cdot u) \cdot d\forall$$

$\because dx \cdot dy \cdot dz = d\forall$

similarly,

$$(\dot{m}_{in} - \dot{m}_{out})_y = -\frac{\partial}{\partial y}(\rho v) \cdot d\forall$$

$$(\dot{m}_{in} - \dot{m}_{out})_z = -\frac{\partial}{\partial z}(\rho w) \cdot d\forall$$

$$(\dot{m}_{in} - \dot{m}_{out}) = -d\forall \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]$$

$$\dot{m}_{st} = \frac{\partial}{\partial t}(m_{st}) \quad \text{- stored mass per sec}$$

$$= \frac{\partial}{\partial t}(\rho \cdot d\forall)$$

$$= d\forall \cdot \frac{\partial \rho}{\partial t}$$

conservation of mass.

$$\dot m_{in} - \dot m_{out} = \dot m_{sto}$$

$$-\cancel{d\forall}\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right] = \cancel{d\forall}\cdot\frac{\partial \rho}{\partial t}$$

$$\boxed{\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)+\frac{\partial \rho}{\partial t}=0}$$

– (Most generalised equation in fluid mechanics as it doesn't involve any assumptions)

Assumptions:

1. Steady flow:

$$\frac{\partial \rho}{\partial t}=0$$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

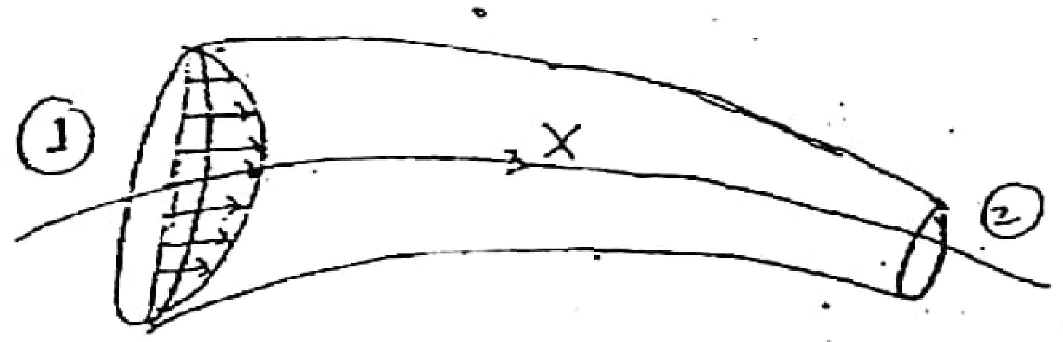
2. Steady and incompressible flow:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \rho = \text{constant}$$

3. 2-Dimensional, steady, incompressible flow.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

4. 1-dimensional, steady flow:



$(\dot m_{in} - \dot m_{out}) = 0$ (since flow is steady

$$\dot m_x - \left(\dot m_x + \frac{\partial \dot m_x}{\partial x}\cdot dx\right) = 0$$

$$\frac{\partial \dot m_x}{\partial x}=0$$

$$\frac{\partial}{\partial x}(\rho\cdot dy\cdot dz\cdot u)=0$$

∵ there is no other c/s area.

∴ $dy\cdot dz = A$ - total area

$u = V$ - velocity of flow

$$\frac{\partial}{\partial x}(\rho\cdot A\cdot V)=0$$

$$(\rho A V) = \text{constant}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \text{constant}$$

If flow is incompressible,

$\rho = \text{constant}$

$$A_1 V_1 = A_2 V_2$$

Q. (Page 36, Q 34)

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)+\frac{\partial \rho}{\partial t}=0$$

$$\rho\cdot\frac{\partial u}{\partial x}+u\cdot\frac{\partial \rho}{\partial x}+\rho\cdot\frac{\partial v}{\partial y}+v\cdot\frac{\partial \rho}{\partial y}+\rho\cdot\frac{\partial w}{\partial z}+w\cdot\frac{\partial \rho}{\partial z}=0$$

$$\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]+2.4=0$$

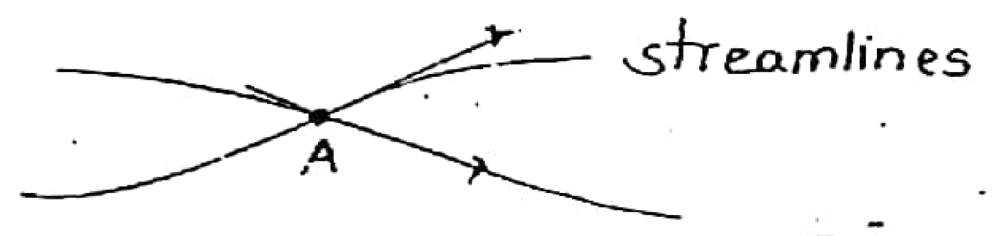
$$\rho\,[\nabla\cdot\bar V] = -2.4$$

$$\nabla\cdot\bar V = \frac{-2.4}{1.2} = -2$$

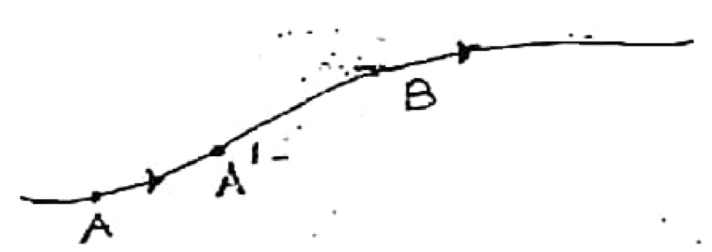
Streamlines :

It is an imaginary line drawn in the flow field in such a way such that tangent drawn at any point on this line directly represents direction of velocity vector of fluid particle at that point.

Two different streamlines can never intersect each other at any one moment.

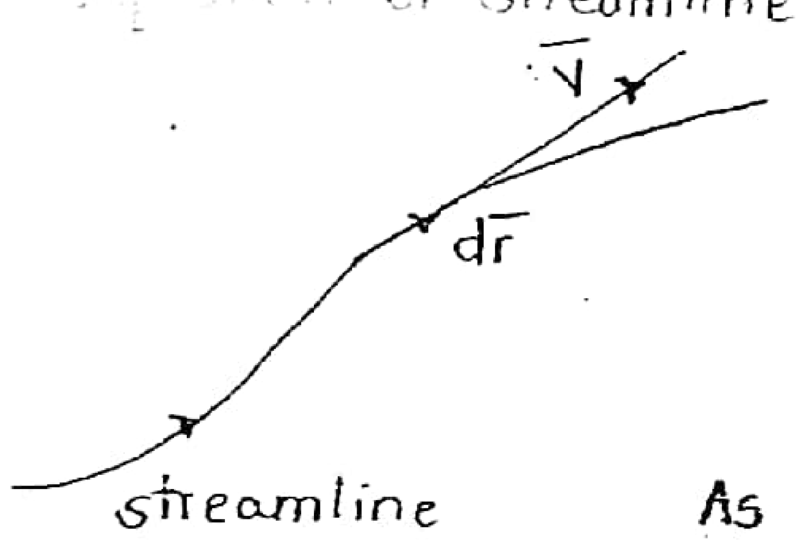


If two streamlines cross each other, particle A will have two different directions at the same point which is not possible. (Particle can have different directions at different points but not at single point).



Streamline give the velocity direction of a particle at that point, but it doesn't give velocity direction of particle when it changes its position (A')

Equation of streamline :



Take a differential position vector along streamline.

$$d\bar{r} = (dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$$

As velocity vector ($\bar{V}$) and position vector ($d\bar{r}$) are along same direction (angle 0)

$$d\bar{r} \times \bar{V} = 0$$

$$\begin{vmatrix} i & j & k \\ dx & dy & dz \end{vmatrix} = 0$$

$$\hat{i}(w\,dy - v\,dz) - \hat{j}(w\,dx - u\,dz) + \hat{k}(v\,dx - u\,dy) = 0$$

$$\hat{i}(w\,dy - v\,dz) = 0$$

$$\frac{dy}{v} = \frac{dz}{w}$$

$$\hat{j}(w\,dx - u\,dz) = 0$$

$$\frac{dx}{u} = \frac{dz}{w}$$

$$\hat{k}(v\,dx - u\,dy) = 0$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Q. A 2-D steady incompressible flow is given by velocity $\bar{V} = (3x\hat{i} - 3y\hat{j})$. Find equ^n of streamline passing through (1,1).

$u = 3x$
$v = -3y$

For 2-D steady incompressible flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 3 + (-3) = 0 \quad \text{(flow is possible)}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\int \frac{dx}{3x} = \int \frac{dy}{-3y}$$

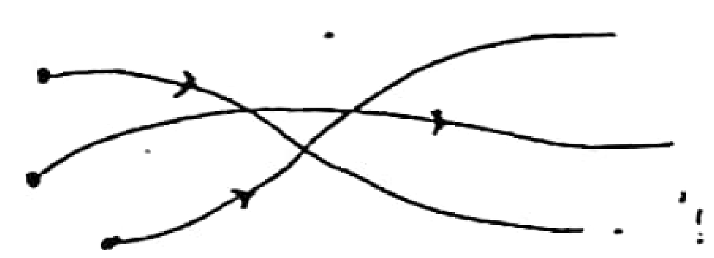
$$\ln x = -\ln y + \ln c$$

$$xy = c$$

$$C = 1 \times 1 = 1$$

Pathlines:

It is an actual path traced by an individual fluid particle.



Pathlines can intersect each other.

Streaklines:

It is a locus of fluid particle at a moment which have been crossed from the same point.

Streamline - direction of velocity vector.

Path lines - Individual motion of fluid particle.

Streak line - Identification of location of fluid particles.

Note: If flow is steady and uniform all three lines will look mathematically similar.

Acceleration of fluid particle:

$\bar{V} = (u\hat{i} + v\hat{j} + w\hat{k})$ — function of (x, y, z, t)

$$d\bar{V} = \frac{\partial \bar{V}}{\partial x} \cdot dx + \frac{\partial \bar{V}}{\partial y} \cdot dy + \frac{\partial \bar{V}}{\partial z} \cdot dz + \frac{\partial \bar{V}}{\partial t} \cdot dt$$

$$\vec{a} = \frac{d\bar{V}}{dt} = \frac{\partial \bar{V}}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial \bar{V}}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial \bar{V}}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial \bar{V}}{\partial t} \cdot \frac{dt}{dt}$$

$$\bar{a} = \underbrace{u \cdot \frac{\partial \bar{V}}{\partial x} + v \cdot \frac{\partial \bar{V}}{\partial y} + w \cdot \frac{\partial \bar{V}}{\partial z}}_{\text{convective acceleration}} + \underbrace{\frac{\partial \bar{V}}{\partial t}}_{\text{Local, temporal}}$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

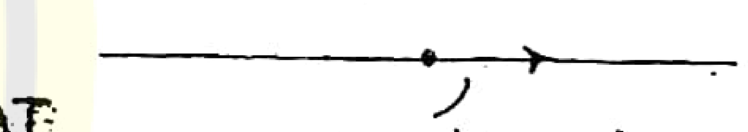
$$a_z = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \text{ units.}$$

If flow is uniform,
convective acceleration $= 0$

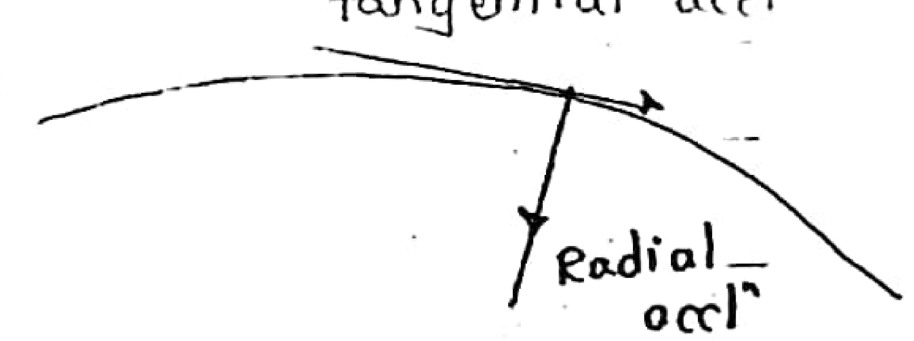
If flow is steady,
Local acceleration $= 0$

Linear motion:



acceleration because of change in magnitude of velocity

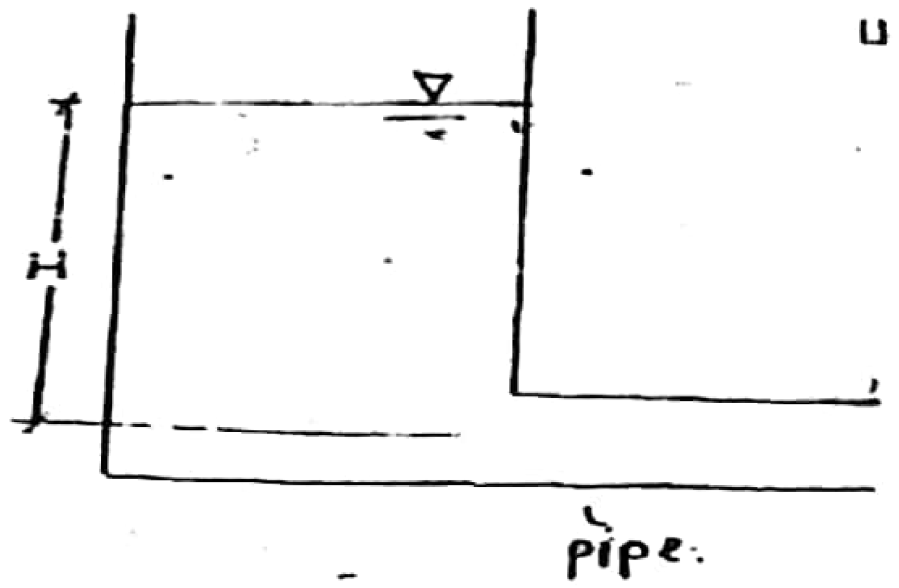
Curvilinear motion



Because of change in direction of velocity - normal accel^n (Centripetal acceleration, radial acceleration - towards the centre.

Because of change in magnitude of velocity - tangential acceleration.

(i) Flow through straight pipe of uniform diameter.



Convective Accel^n = 0 (c/s = const)

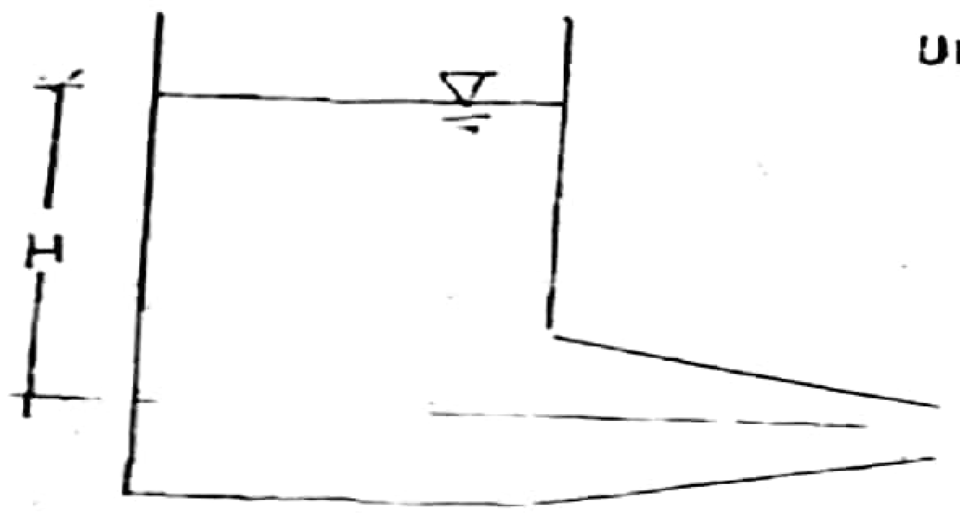
If H = constant (steady flow)

Local accl^n = 0

If H = variable (unsteady flow)

Local accl^n $\neq 0$

(ii) Flow through straight pipe of non-uniform diameter.



Convective accl^n $\neq 0$ (c/s changing)

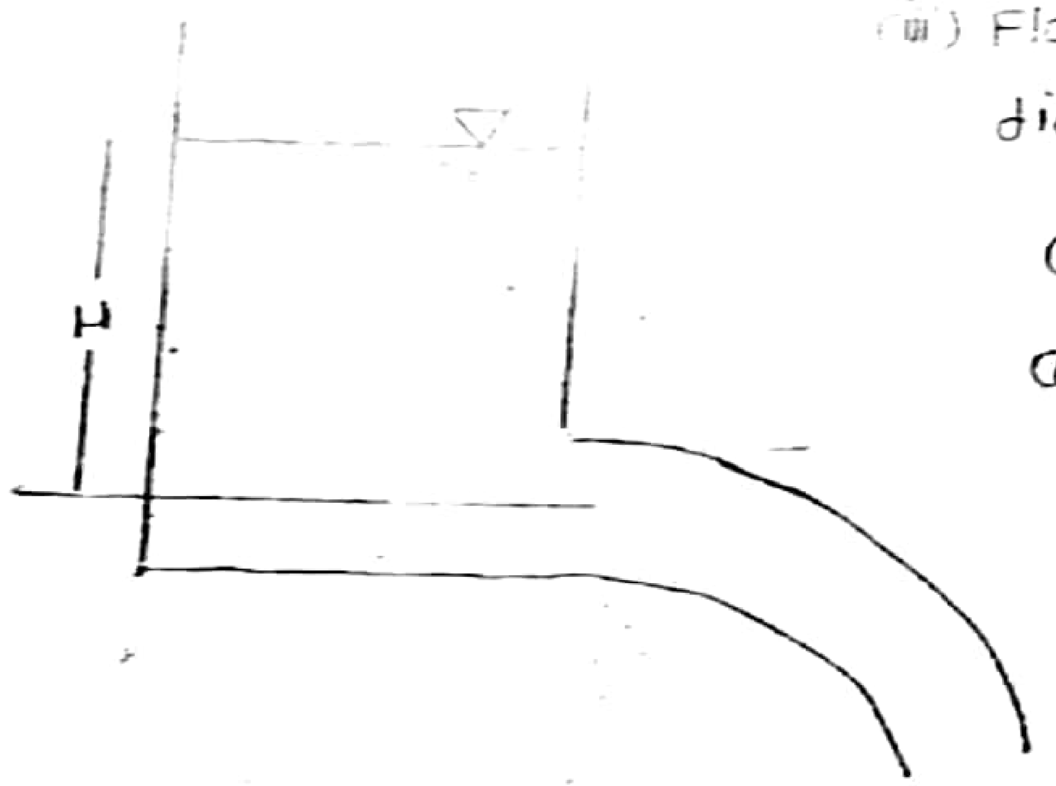
If H = constant (steady flow)

local accl^n = 0

If H = variable (unsteady flow).

Local accel^n $\neq 0$

(iii) Flow through curved pipe of uniform diameter.



Convective tangential accl^n = 0

convective radial accl^n $\neq 0$

If H = constant (steady flow)

Local tangential accl^n = 0

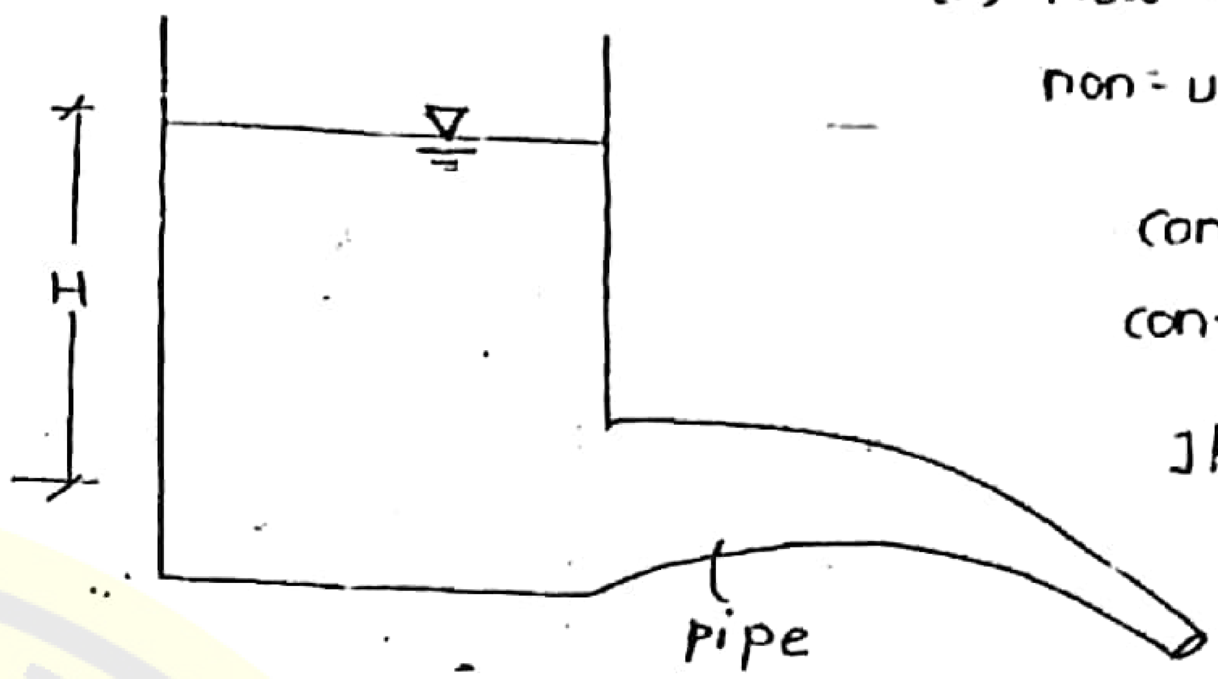
Local radial accl^n = 0

If H = variable (unsteady flow)

Local tangential accl^n $\neq 0$

Local radial accl^n = 0

(iv) Flow through curved pipe of non-uniform diameter



conventional tangential accl^n $\neq 0$

conventional normal accl^n $\neq 0$

If H = constant (steady)

Local tangential accl^n = 0

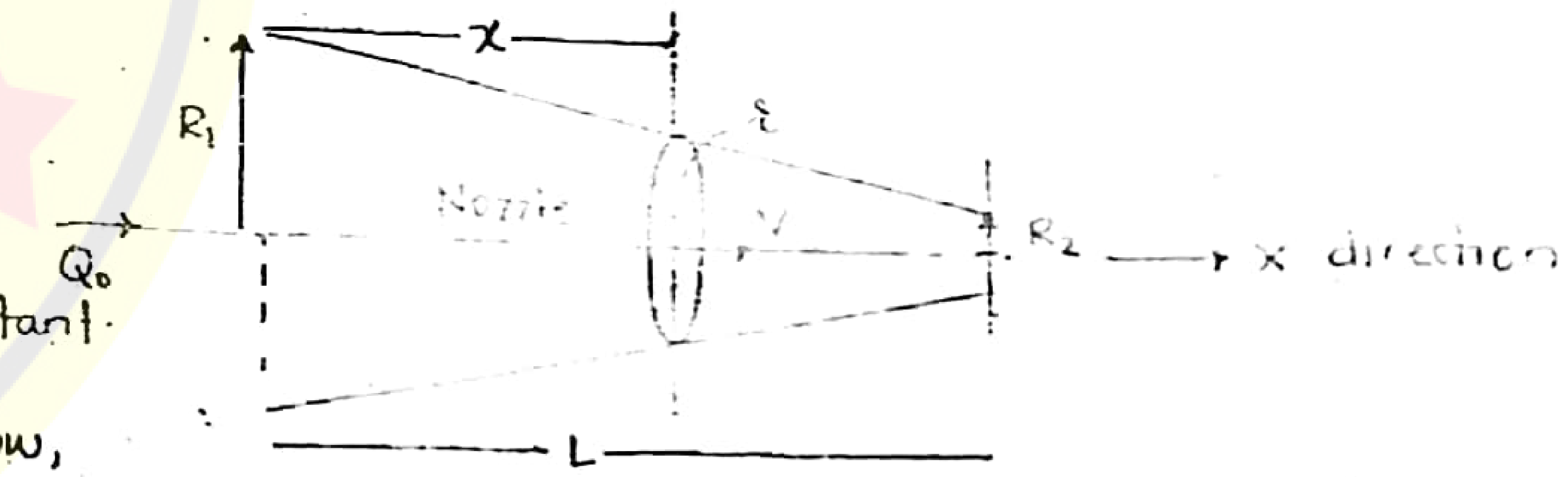
Local radial accl^n = 0

If H = variable (unsteady)

Local ~~tangential~~ tangential accl^n $\neq 0$

Local radial accl^n = 0

Q. Find the acceleration of fluid at exit section.



constant

steady flow, constan head.

At the section x-x.

$$a = a_{\text{convective}}$$

$$= V \cdot \frac{\partial V}{\partial x} \quad (\text{unidirectional flow})$$

$$= \frac{Q_0}{A} \cdot \frac{\partial}{\partial x}\left(\frac{Q_0}{A}\right)$$

$$\frac{R_1 - R_2}{L} = \frac{R_1 - r}{x}$$

$z = R_1 - Bx$ $\quad$ $B = \left(\frac{R_1 - R_2}{L}\right)$

$$a = \frac{Q_0}{\pi(R_1 - Bx)^2} \cdot \frac{\partial}{\partial x}\left(\frac{Q_0}{\pi(R_1 - Bx)^2}\right)$$

$$= \frac{Q_0^2}{\pi^2 (R_1 - Bx)^2} \frac{\partial}{\partial x}\left(\frac{1}{(R_1 - Bx)^2}\right)$$

$$= \frac{2B \cdot Q_0^2}{\pi^2 (R_1 - Bx)^5}$$

At exit section,
$$a = \frac{2B \cdot Q_0^2}{\pi^2 (R_1 - BL)^5}$$

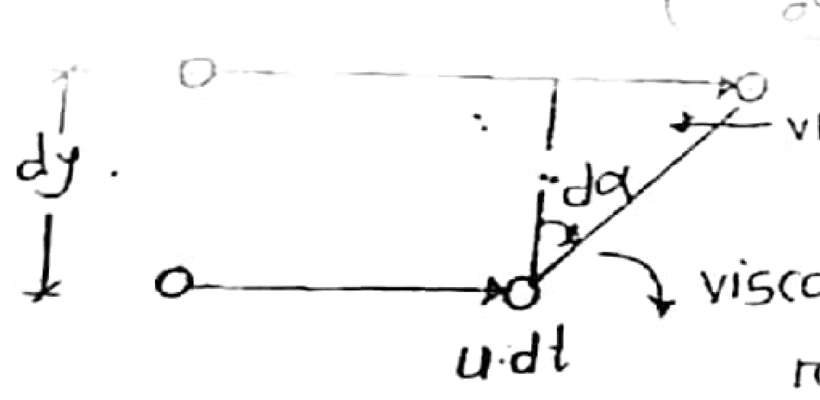
Rotational components of fluid :

↻ − ve $\qquad$ ↺ + ve

In XY plane,

$$\vec{V} = (u\hat{i} + v\hat{j})$$

(i) u-shore of rotation

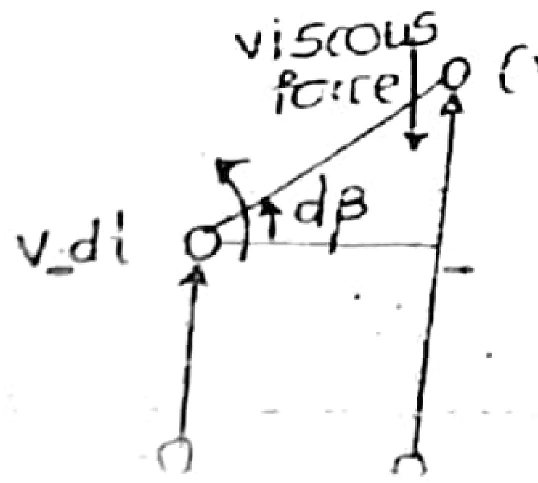


$$\tan d\alpha = d\alpha = \frac{-\frac{\partial u}{\partial y} dy \cdot dt}{dy}$$

$$d\alpha = -\frac{\partial u}{\partial y} \cdot dt$$

$$\frac{d\alpha}{dt} = -\frac{\partial u}{\partial y}$$

(ii) v-shore of rotation :



$$\tan d\beta = d\beta = \frac{\frac{\partial v}{\partial x} \cdot dx \cdot dt}{dx}$$

$$= \frac{\partial v}{\partial x} \cdot dt$$

∴ Angular velocity of fluid particle about z-axis

$$\omega_z = \frac{d\beta}{dt} + \frac{d\alpha}{dt}$$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

— since the obtained rotation is effect of 2-particles (divide by 2)

In general in 3-D flows,

$$\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V})$$

$$\vec{\omega} = \frac{1}{2}\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$$

$$= \frac{1}{2}\left[i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\right] - j\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) + k\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial y}\right)]$$

$$= \frac{1}{2}\left[\omega_x - \omega_y + \omega_z\right]$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

If $\omega_x = \omega_y = \omega_z = 0$
$\vec{\omega} = 0$ $\quad$ irrotational flow.

If any of ω_x or ω_y or $\omega_z \neq 0$, —
$\vec{\omega} \neq$ $\quad$ — rotational flow

For 2-D flow

If $\omega_z = 0$ — irrotational flow
$\omega_z \neq 0$ — rotational flow.

Vorticity :

It is rotational strength in the pair of fluid particles. (angular velocity between two particles)

$$2\vec{w} = (\nabla \times \vec{V})$$

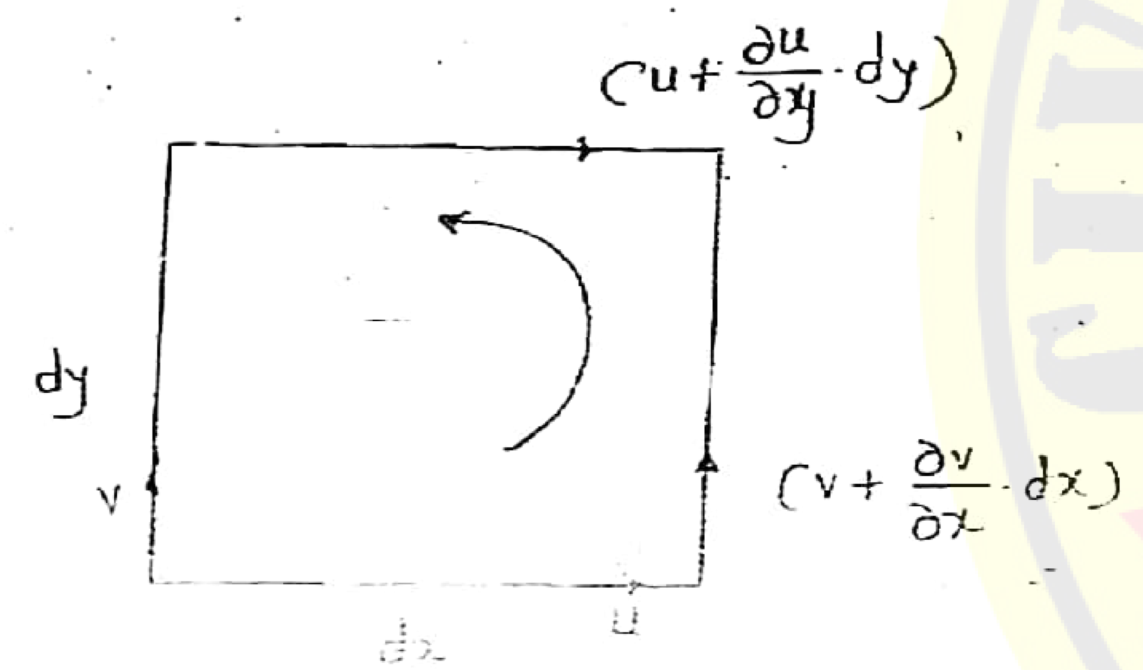
Circulation (Γ)

It is strength of rotation in fluid zone.

It is defined as the line integral of velocity vector taken along a closed loop.

$$\Gamma = \oint \vec{V} \cdot \vec{dr}$$ line - dot product

For a differential (very small) system



$$\Gamma = \oint \vec{V} \cdot \vec{dr} = u \cdot dx + \left(v + \frac{\partial v}{\partial x} \cdot dx\right) \cdot dy - \left(u + \frac{\partial u}{\partial y} \cdot dy\right) \cdot dx - v \cdot dy$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx \cdot dy$$

$$= 2 w_z \cdot A$$

A - area of differential zone. valid for diff. system

Γ = vorticity x area of zone -

It is applicable only when vorticity is constant. If vorticity

Q. (Page 26, Q.38)

z = 2 units.

$u = 2x + 3y$

$v = -2y$.

vorticity $= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$= 0 - 3$

$= -3$ — constant

$\Gamma = -3 \times \pi z^2$

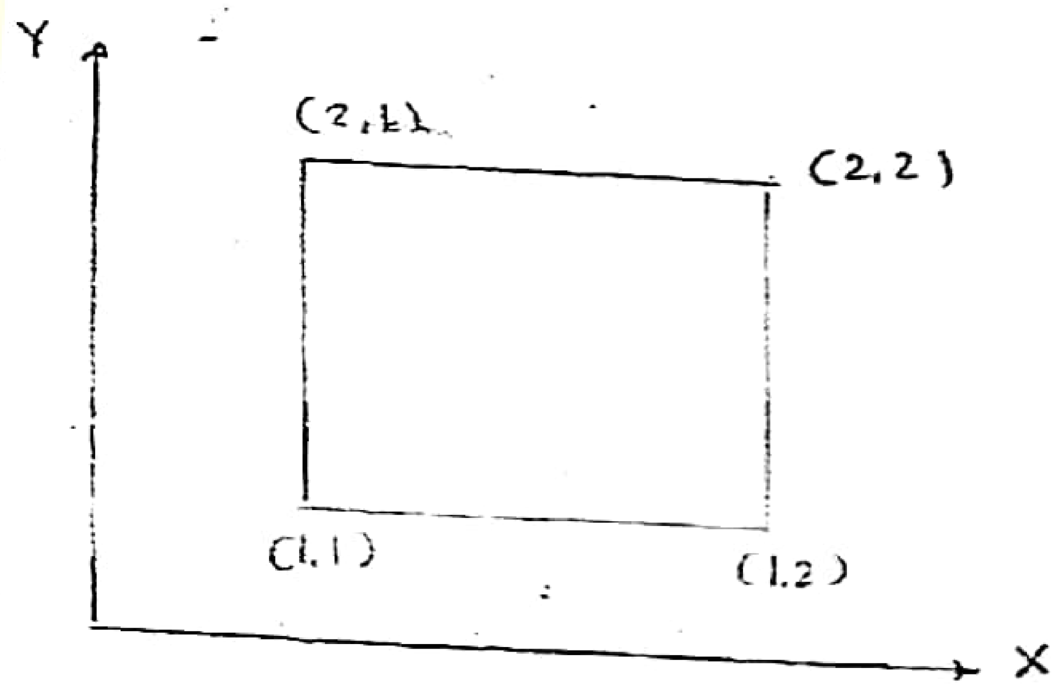
$= -3 \times \pi \times (2)^2$

$= -12\pi$ i.e. clockwise

Q. (Page 29, Q 9)

$u = x^2$

$v = 2xy$.



$$\Gamma = \oint \vec{V} \cdot \vec{dr} = \int_1^2 x^2 \cdot dx + \int_1^2 2(2y) \cdot dy + \int$$

$= 3$ units.

Velocity potential function (ϕ)

It is defined as function of space and time in such a way that negative derivative of this function w.r.t. space directly gives velocity in that direction.

By definition

$$-\frac{\partial \phi}{\partial x} = u$$

$$-\frac{\partial \phi}{\partial y} = v$$

$$-\frac{\partial \phi}{\partial z} = w$$

Boundation of ϕ.

For 2-D steady, incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ — for flow to occur. - (continuity equ^n)

$$\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial y}\right) = 0$$

$$-\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

i.e. $\nabla^2 \phi = 0$

Laplacian operator

ϕ must satisfy the Laplace's equation

Physical significance of ϕ:

Angular velocity of particle.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial x}\right)\right]$$

$$= \frac{1}{2}\left[\frac{-\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x}\right]$$

$$\omega_z = 0$$

i.e. flow is irrotational.

Velocity potential function (ϕ) only exists for the irrotational flow.

Equipotential lines:

(ϕ - constant)

It is line joining the points of equal potential function values.

For steady incompressible flow

$$\phi = f(x, y)$$

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$$

$$d\phi = -u \cdot dx - v \cdot dy$$

For equipotential lines,

$$\phi = \text{constant}.$$

$$d\phi = 0$$

$$-u \cdot dx - v \cdot dy = 0$$

$$\frac{dy}{dx} = \frac{-u}{v}$$ — slope of equipotential line

Stream function (ψ)

In general, this function is defined as function of space and time in such a way such that continuity equation is satisfied.

By definition.

$$u = -\frac{\partial \psi}{\partial y}$$

$$v = +\frac{\partial \psi}{\partial x}.$$

By continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right)$$

$$= \frac{-\partial \psi}{\partial x \cdot \partial y} + \frac{\partial \psi}{\partial x \cdot \partial y}$$

$$= 0 \quad \text{- (satisfied automatically)}$$

ψ exists in rotational as well as irrotational flow.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

$$= \frac{1}{2}\nabla^2 \psi$$

If ψ satisfies Laplace's equ^n

i.e. $\nabla^2 \psi = 0$ - irrotational flow ($\omega = 0$).

and if $\nabla^2 \psi \neq 0$

Equi-stream function lines.
(ψ - constant line).

It is the line joining the points having same stream function value.

For 2-D steady, incompressible flow.

$$\psi = f(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

$$= (v \cdot dx - u\, dy)$$

For equi stream function lines, ψ = constant

$$d\psi = 0$$

$$v \cdot dx - u\, dy = 0$$

$$\frac{dy}{dx} = \frac{v}{u} \quad \text{— slope of equi-stream function line}$$

$\frac{dy}{v} = \frac{dx}{u}$ is equation of streamline.

$\therefore$ Stream lines are the only equi-stream function lines.

$$\left(\frac{dy}{dx}\right)_{\phi\text{-const line}} \times \left(\frac{dy}{dx}\right)_{\psi\text{ const line}} = \frac{-u}{v} \times \frac{v}{u}$$

$$= -1$$

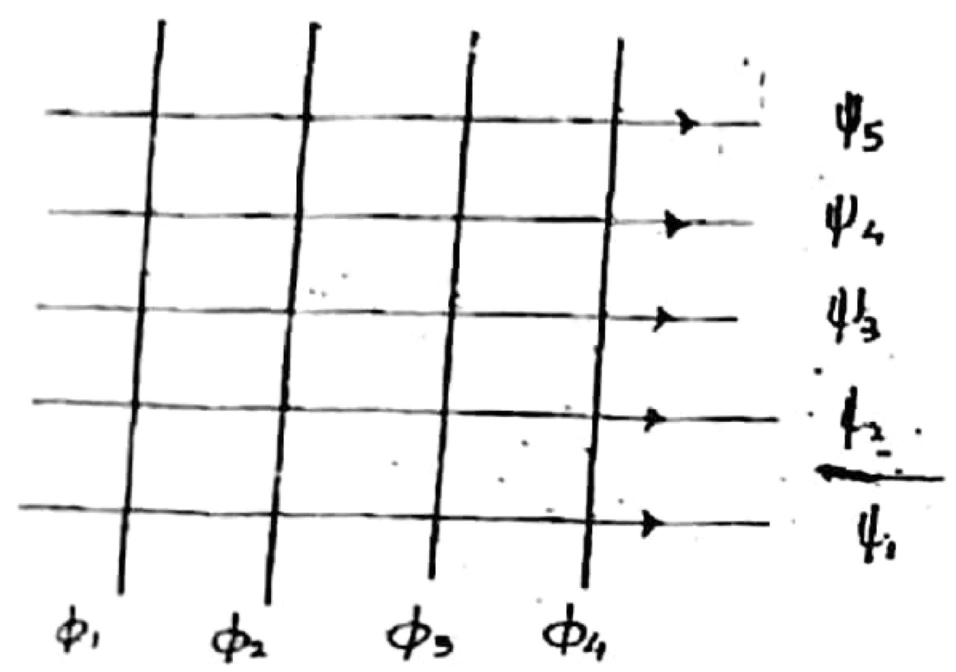
Equipotential lines and streamlines in irrotation flow are always orthogonal to each other.

A graphical diagram representing equipotential lines and streamlines in an irrotational flow is known as flow net.

Thursday
21st Nov' 2013

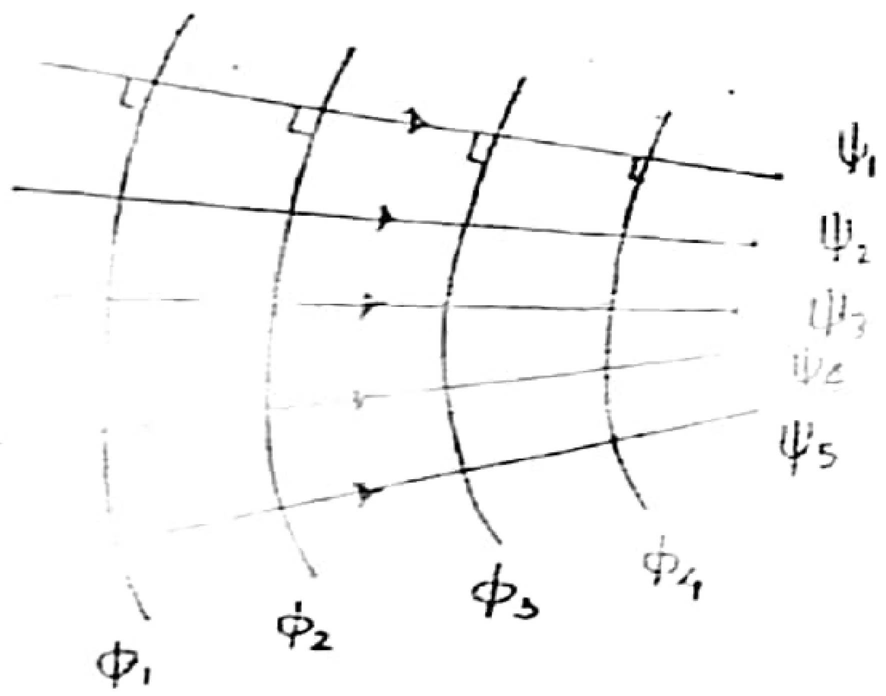
Flow nets in different Flows :

(i) Uniform Flows :



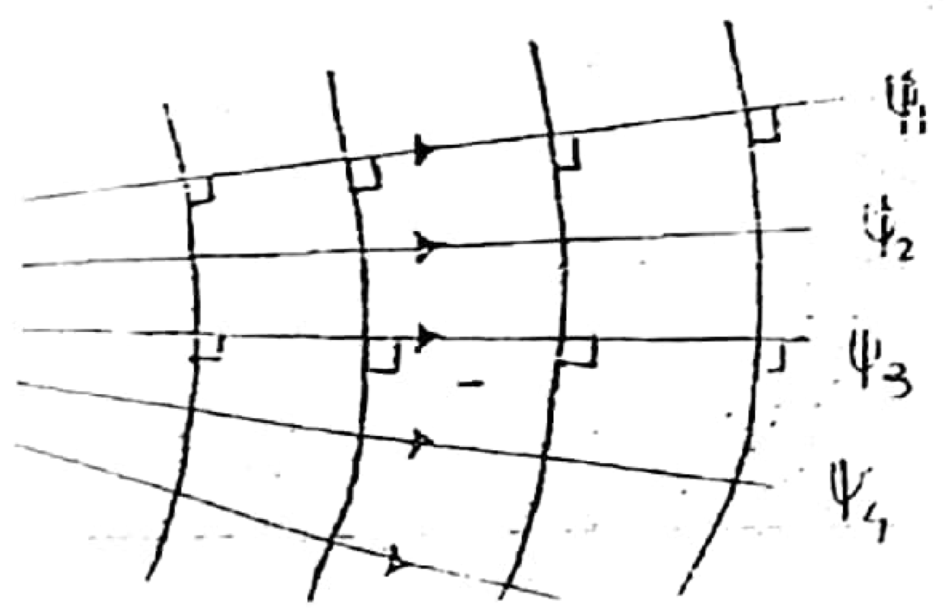
(ii) Accelerated flow :

(streamlines are converging)



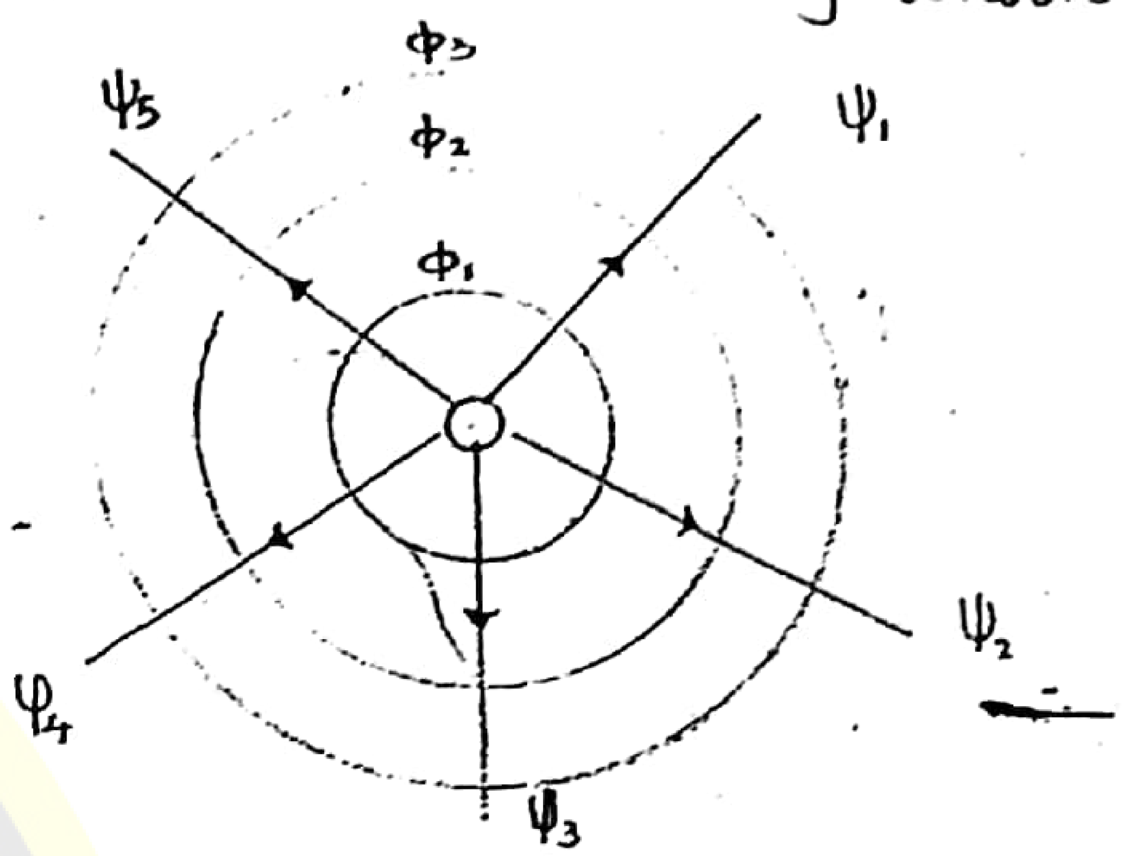
(iii) Retarding Flow :

(streamlines are diverging)



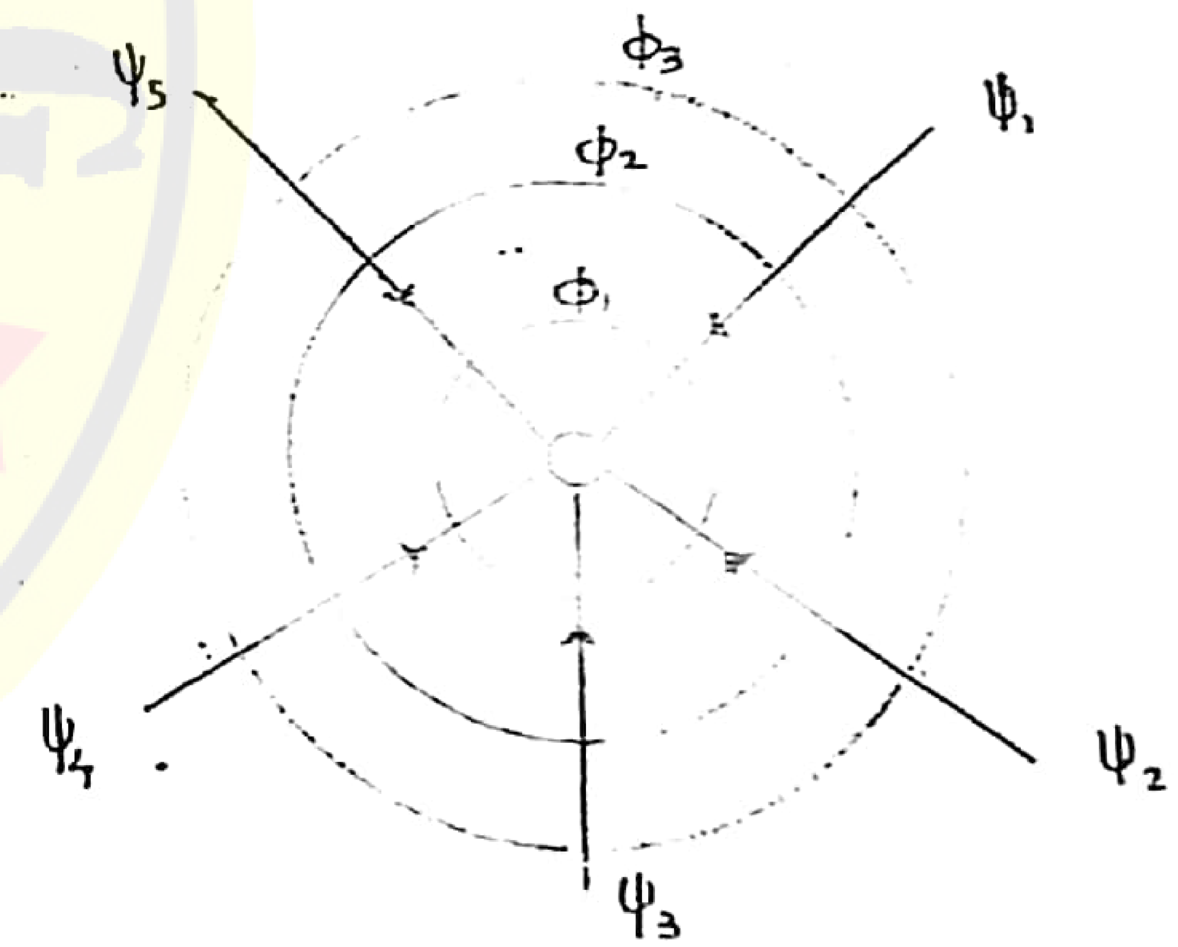
(iv) Source flow :

(streamlines move radially outward)

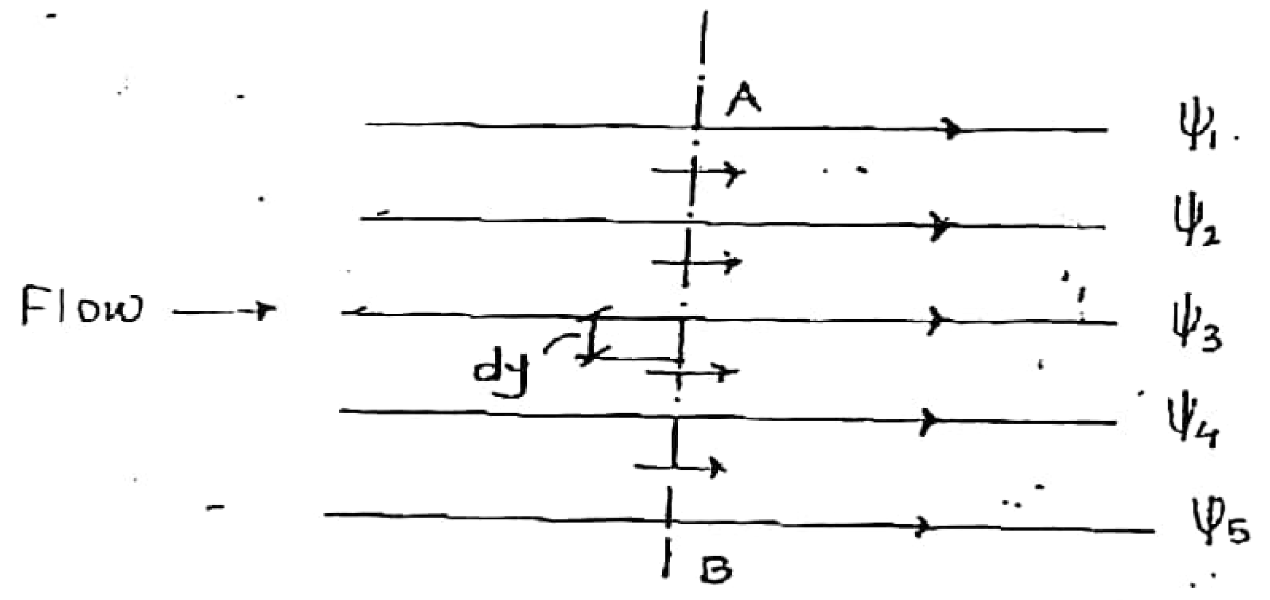


(v) Sink :

(streamlines move radially inward.)



Physical significance of stream function (ψ):



In 2-D, steady incompressible flow,

$$\psi = f(x, y)$$

$$d\psi = \cancel{\frac{\partial \psi}{\partial x} dx}^{\,0} + \frac{\partial \psi}{\partial y} \cdot dy$$

(no distance in x-direction)

$$d\psi = \frac{\partial \psi}{\partial y} \cdot dy$$

$$= -u \cdot dy$$

$$-d\psi = u \cdot dy$$

$$= u \cdot (dy \times 1)$$

- unit width (depth) of flow

$$\int_A^B -d\psi = \int dQ \text{ (per unit width of flow)}$$

$$\Delta \psi = Q \text{ per unit width of flow.}$$

i.e.

the difference of stream function value between any two points in flow gives the discharge per unit width of flow between these two points.

Q.

Page 24, Q-18

Q. A 2-D steady, incompressible flow is given by

$$\vec{V} = (3xy)\,i + \left(-\frac{3}{2}x^2 \pm \frac{3}{2}y^2\right) j$$

Find the relevant potential and stream function.

$$u = 3xy$$

$$v = \left(-\frac{3}{2}x^2 \mp \frac{3}{2}y^2\right)$$

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 3y + (0 - \cancel{3y})$$

$$= 0 \quad \text{- Flow is possible.}$$

For potential function (ϕ)

Angular velocity (ω_z) -

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left(3x - 3x\right)$$

$$= 0 \quad \text{- Irrotational flow}$$

ϕ will exist for given flow

$$u = -\frac{\partial \phi}{\partial x}$$

$$\frac{\partial \phi}{\partial x} = -u = -3xy$$

$$\partial \phi = -3xy \; \partial x$$

$$\int \partial \phi = -y \int 3x \, \partial x$$

$$\phi = -\frac{3x^2}{2} y + f(y) + c$$

There can be pure

similarly, $v = \frac{-\partial\phi}{\partial y}$

$$\frac{\partial\phi}{\partial y} = -v = \frac{3}{2}y^2 - \frac{3}{2}x^2$$

$$\int \partial\phi = \int \left(\frac{3}{2}y^2 - \frac{3}{2}x^2\right)\partial y.$$

$$\phi = \frac{-3}{2}\cdot\frac{y^3}{3}x^2 + \frac{y^3}{2} + f(x) + c \quad \text{--- (ii)}$$

There can be pure function of x.

from — (i) and — (ii)

$$\phi = \frac{-3}{2}x^2 y + \frac{y^3}{2} + c.$$

For stream function

$$u = -\frac{\partial\psi}{\partial y}$$

$$\int \partial\psi = \int -3xy \cdot \partial y$$

$$\psi = \frac{-3x}{2}y^2 + f(x) + c \quad \text{--- (iii)}$$

There can be pure function of x.

and $v = \frac{\partial\psi}{\partial x}$

$$\int \partial\psi = \int \left(\frac{3}{2}x^2 - \frac{3}{2}y^2\right)\cdot\partial x$$

$$\psi = \frac{x^3}{2} - \frac{3}{2}y^2\cdot x + f(y) + c \quad \text{--- (iv)}$$

from — (iii) and — (iv)

$$\psi = \frac{-3xy^2}{2} + \frac{x^3}{2} + c$$

Cauchy - Riemann equation
(Irrotational flow)

$$u = \frac{-\partial\phi}{\partial x} = \frac{-\partial\psi}{\partial y} \quad \text{--- (i)}$$

&

$$v = \frac{-\partial\phi}{\partial y} = +\frac{\partial\psi}{\partial x} \quad \text{--- (ii)}$$

from — (i) & — (ii)

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$$

and

$$\frac{-\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$$

Note:

(i) Angular velocity $(\omega_z) = \dfrac{\left(\frac{d\beta}{dt} + \frac{d\alpha}{dt}\right)}{2} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

(vector quantity)

(ii) Rate of shear deformation $= \dfrac{\left|\frac{d\beta}{dt}\right| + \left|\frac{d\alpha}{dt}\right|}{2}$

(scalar quantity)

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$